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A symmetry measure for damage detection with mode shapes

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ABSTRACT

This paper introduces a feature for detecting damage or changes in structures, the continuous symmetry measure, which can quantify the amount of a particular rotational, mirror, or translational symmetry in a mode shape of a structure. Many structures in the built environment have geometries that are either symmetric or almost symmetric, however damage typically occurs in a local manner causing asymmetric changes in the structure's geometry or material properties, and alters its mode shapes. The continuous symmetry measure can quantify these changes in symmetry as a novel indicator of damage for data-based structural health monitoring approaches. This paper describes the concept as a basis for detecting changes in mode shapes and detecting structural damage. Application of the method is demonstrated in various structures with different symmetrical properties: a pipe cross-section with a finite element model and experimental study, the NASA 8-bay truss model, and the simulated IASC-ASCE structural health monitoring benchmark structure. The applicability and limitations of the feature in applying it to structures of varying geometries is discussed.

1. Introduction

Structural mode shapes have been commonly used as a feature for structural health monitoring (SHM). Methods such as mode shape difference, slope, and curvature are ways to detect damage in beam-like structures using mode shapes and frequency response function data [1–3]. Direct comparisons of the differences in mode shapes work well in laboratory testing situations, however they may not be particularly informative in the field due to measurement noise or environmental effects. For mode shape curvature, prior knowledge of the structure may not be necessary as long as it is similar to a beam; damage is approximated as a discontinuous change in the *EI* bending stiffness term of the beam equation, which also causes a discontinuity in the second derivative of the mode shape, also known as the mode shape curvature. Mode shape slope and curvature are also susceptible to errors due to numerical differentiation and uncertainty in modal identification of frequencies and mode shape coordinates [4]. When structures are more complex beyond that of a simple beam or those that can be approximated as a beam such as a single span bridge [5], these methods tend to be more difficult to use. Other mode shape methods such as modal strain energy require a detailed model of the structure which limits the situations in which they can be used [6].

Data-based methods for SHM eschew structural models for an approach that uses data from sensors in a presumed healthy state of the structure as a baseline and looks for statistically significant differences in features extracted from the measured structural responses for indications of change or damage to the structure [7]. Examples of data-based methods of

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damage detection include ones based on statistical process control [8] and the one-class support vector machine [9]. A key requirement for these statistical methods is that the features are indicative of damage or change in the structure, otherwise the statistical methods would not work. Features can be based on signal characteristics or physics-based information about the structure, and generally a collection of varied features is necessary for detection accuracy.

Symmetry is an intrinsic property of many natural and engineered structures. Objects that are symmetric tend to be aesthetically pleasing and may also be the best design for a structure, as an asymmetry might introduce uneven loading or stresses. Previous work has made use of axisymmetric vibration modes or guided waves in pipes for detection of notches and corrosion damage in pipe systems, as guided waves would reflect off any defects in the pipe wall [10,11]. Most damage to structures usually occurs locally to elements of the structure, and generally does not affect their symmetric counterparts elsewhere in the structure. Asymmetric damage in the structure then alters its mode shapes and may cause them to be more asymmetrical. Regarding vibrational behavior, asymmetry in geometry, stiffness, and mass might introduce vibration modes that are more likely to be coupled or closely spaced. A measurement of how symmetric or asymmetric a structure's mode shapes are, could be a useful indication of an asymmetric or local change in the structure's geometrical or physical properties. However, in the case of highly asymmetric or irregular structures, this information would not be particularly useful.

In this paper, we demonstrate the use of a symmetry metric as a feature for damage detection in structures. The continuous symmetry metric (CSM) was introduced by Zabrodsky et al. for the identification of chemical compounds [12], and image analysis in computer vision [13]. We adapt its use to measure the amount of symmetry present in the mode shapes of a structure, and use it as an indication of damage or change in the structure. It adds to the toolbox of features for detecting damage using mode shapes, specifically in the case where the structure exhibits some form of symmetry. We explain how to calculate the CSM, and demonstrate it on three structures of different topologies: pipes with both a finite element model and an experimental study with intact and corroded pipe sections, the NASA 8-bay truss, and the simulated IASC-ASCE SHM benchmark structure. Finally, we discuss how the feature can be applied in varying types of structures and its limitations.

2. Continuous symmetry measure

The CSM is a way to quantify how symmetric a given shape is. The method was initially proposed for rotational symmetry of polygons of N-sides [12], but it can easily be extended for any kind of rotational, translational, or mirror symmetry, and applied to measure the amount of symmetry in the mode shapes of a structure. The amount of symmetry in the mode shapes will depend on how symmetric the geometry of a structure is, as well as the mass, and stiffness properties. As damage will likely alter these properties asymmetrically, for an otherwise symmetric structure, an increase in the CSM could be indicative of damage in the structure, and could be used as a feature for SHM.

As a note, the formal mathematical representation of a rotation symmetry where N rotations are made to return a shape to its original configuration is described as cyclic group C_N, while a mirror symmetry is described as cyclic group C_S. For more information about cyclic groups and group theory the reader is referred to [14].

2.1. Derivation

Given an N-sided polygon (Fig. 1a) that we want to calculate the CSM for cyclic group C_N with rotational symmetry, the methodology is as follows, as shown in Fig. 1. First, the center of mass c_m of the polygon is found (Fig. 1b), and the polygon is normalized such that the furthest vertex has a distance of 1. Then a C_N symmetrized version of the original polygon is calculated, by first rotating the vertices n by $-n\frac{2\pi}{N}$ radians, moving them into the same $\frac{2\pi}{N}$ slice (Fig. 1c) to make a folded shape, averaging their position (Fig. 1d), and rotating the vertices back. This creates a C_N symmetrized version of the polygon (Fig. 1e). The mean squared distance of each vertex v from it's corresponding symmetrized vertex v' is found and this value is the continuous symmetry measure as shown in Eq. (1). It has a range of 0 to 1, with 0 representing perfect symmetry, and 1 perfect asymmetry (see the appendix of [12] for the derivation).

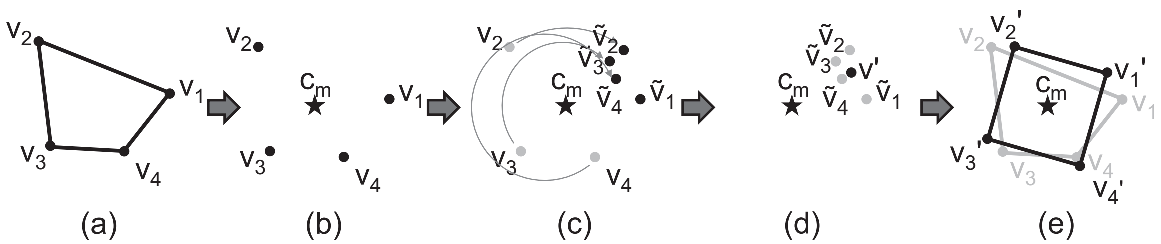

Fig. 1. Workflow for calculating the CSM: (a) The original N-sided polygon. (b) Normalizing the size of the polygon and finding the center of mass c_m. (c) Rotating vertices by multiples of $\frac{2\pi}{N}$ to so that they are all in the same $\frac{2\pi}{N}$ slice for the folded shape. (d) Finding the average v' of the rotated vertices. (e) The symmetrized polygon used to calculate the CSM.