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Dimension reduction of Karhunen-Loeve expansion for simulation of stochastic processes

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ABSTRACT

Conventional Karhunen-Loeve expansions for simulation of stochastic processes often encounter the challenge of dealing with hundreds of random variables. For breaking through the barrier, a random function embedded Karhunen-Loeve expansion method is proposed in this paper. The updated scheme has a similar form to the conventional Karhunen-Loeve expansion, both involving a summation of a series of deterministic orthonormal basis and uncorrelated random variables. While the difference from the updated scheme lies in the dimension reduction of Karhunen-Loeve expansion through introducing random functions as a conditional constraint upon uncorrelated random variables. The random function is expressed as a single-elementary-random-variable orthogonal function in polynomial format (non-Gaussian variables) or trigonometric format (non-Gaussian and Gaussian variables). For illustrative purposes, the simulation of seismic ground motion is carried out using the updated scheme. Numerical investigations reveal that the Karhunen-Loeve expansion with random functions could gain desirable simulation results in case of a moderate sample number, except the Hermite polynomials and the Laguerre polynomials. It has the sound applicability and efficiency in simulation of stochastic processes. Besides, the updated scheme has the benefit of integrating with probability density evolution method, readily for the stochastic analysis of nonlinear structures.

1. Introduction

Engineering excitations such as earthquakes, wind and waves are of the stochastic processes in nature. Due to the essential non-stationarities of stochastic excitations and the significant nonlinearities of structural performance under these actions, the conventional spectrum-transfer schemes in frequency domain for structural stochastic analysis are not applicable. In this case, the power spectral density of stochastic excitations needs to be decomposed into sample processes in time domain for accurate stochastic response and reliability analysis of nonlinear structures.

Owing to the simple principles and ready-to-implement algorithm, random simulation method of stochastic processes receive great appeals in the past decades. Representative random simulation methods include (a) linear filter method [1], i.e., autoregressive (AR), moving average (MA) and autoregressive moving average (ARMA); (b) spectral representation

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method (SRM) [2]; (c) proper orthogonal decomposition (POD) [3], which is also referred to as the Karhunen-Loeve expansion [4]. In general, linear filter method has the ability of approximating the target spectral density matrix ultilizing a few parameters. While the implementation of algorithm is complicated, involving the estimation of categories, orders and parameters of models. It is thus limited in engineering applications in some cases. The spectral representation method has advantages of simple algorithm, rigorous theory and reliable simulation. This method, however, has to deal with a high-dimensional issue invovling a large number of random variables. In order to reduce the number of random variables, some efforts on the functional-set definition with respect to random variables from the first family of spectral representation [5] and on the random optimization with respect to frequencies and phase angles from the second family of spectral representation [6–8] were proceeded, respectively.

Power spectral density function is the critical component of spectral representation method, whereby the spectral representation method merely requires a fast Fourier transform (FFT) to implement the simulation of stochastic processes. While since the complete relationship between the spectral density function and the finite samples of stochastic processes is not fairly revealed, hundreds of terms need to be retained in order to accommodate the range of frequency and the uncertainty of phase angles, especially for the stochastic processes with short-time correlation. Consequently, it is appropriate to represent the randomness of stochastic processes by elementary variables so that the connection between random functions and sample processes could be readily established [9,10]. This thought is fully included in the Karhunen-Loeve expansion.

As a matter of fact, the Karhunen-Loeve expansion has a similar theoretical basis with the spectral representation method [5,11]. Both of them represent the stochastic processes as a summation of a series of deterministic functions modulated by a row of unrelated random coefficients (random variables). Compared with the spectral representation method, the Karhunen-Loeve expansion requires the solution of an eigenvalue problem [12]. It does not, of course, require the existence of a spectral density function, i.e., it is ready-made for non-stationary processes. However, the method is limited in practical application due to the difficulty of solving Fredholm integral equation. It still incurs, meanwhile, large number of random variables for the simulation of stochastic processes though the Karhunen-Loeve expansion has the optimality in the 2-norm sense. In order to solve the problem, some orthogonal expansion methods based on different orthonormal basis, e.g., generalized polynomial chaos [13], radial basis functions [14], and wavelet basis [15,16], were proposed. They are rigorously equivalent to the conventional Karhunen-Loeve expansion when the term number $N \to \infty$ [17,18]. In this manner, however, the number of random variables of orthogonal functions involved in a complete series expansion is still large.

An updated scheme of spectral representation was proposed through introducing the random functions as a conditional constraint [5]. This treatment bypasses the challenge of high-dimensional matter inherent in the conventional spectral representation. Moreover, the accuracy of simulation in the updated scheme can be improved through increasing the numbers of both expansion terms and representative samples. It was demonstrated that using the updated scheme with single-variable random functions, the accurate second-order statistics can be secured. Numerical investigations relevant to the simulation of stationary and non-stationary seismic acceleration processes demonstrated the applicability and efficiency of the method [5].

In this paper, a random function embedded Karhunen-Loeve expansion method for representation and simulation of stochastic processes is proposed. The updated scheme has a similar form to the conventional Karhunen-Loeve expansion, where the stochastic process is represented by a summation of a series of deterministic orthonormal basis and orthonormal random variables. A collection of random functions is introduced so as to implement the dimension reduction of discrete representation of stochastic processes. The remaining sections arranged in this paper are distributed as follows. Section 2 revisits the Karhunen-Loeve expansion for simulation of stochastic processes. The random function representation of kernel variables for Karhunen-Loeve expansion is detailed in Section 3, including a family of uncorrelated non-Gaussian variables and a family of uncorrelated Gaussian variables. Implementation procedure and error definition of the updated scheme are addressed in Section 4. For illustrative purposes, the simulation of random seismic ground motions is included in Section 5. Utilizing the probability density evolution method, case study on response analysis of randomly based-excited nonlinear structures is carried out in Section 6. The concluding remarks are included in Section 7.

2. Karhunen-Loeve expansion of stochastic processes

A physically based stochastic process $X(t)$ is a real-valued process, which is defined on the probability space $(\Omega, \mathcal{F}, P)$ and indexed on the bounded interval $[0, T]$. Without loss of generality, the stochastic process with a zero-mean and finite variance is investigated, where $E[X(t)] = 0$ and $E[X^2(t)]$ is bounded for all $t \in [0, T]$. The stochastic process can be approximately represented by a finite series of Karhunen-Loeve (K-L) expansion [19]

$$X(t) \doteq \sum_{i=1}^{N} \sqrt{\lambda_i}\,\xi_i f_i(t) \tag{1}$$

where $\{\xi_i\}_{i=1}^{N}$ denotes a set of uncorrelated standardized variables; N denotes the Karhunen-Loeve expansion terms; λ_i and