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Optimal parameters uncoupling vibration modes of oscillators

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ABSTRACT

This paper proposes a novel optimization concept for an oscillator with two degrees of freedom. By using specially defined motion ratios, we control the action of springs to each degree of freedom of the oscillator. We aim at showing that, if the potential action of the springs in one period of vibration, used as the payoff function for the conservative oscillator, is maximized among all admissible parameters and motions satisfying Lagrange's equations, then the optimal motion ratios uncouple vibration modes. A similar result holds true for the dissipative oscillator having dampers. The application to optimal design of vehicle suspension is discussed.

1. Introduction

In engineering practice a vibration isolator is often required to reduce the transmission of forces or displacements to special bodies, mountings, or bearings while the system is excited. If the vibration of the bodies remains small and well controlled around a desired position of equilibrium for most of excitations, a comfortable, light, and durable system is created. The optimal design of vibration isolator can then be realized depending on the specific goal expressed in terms of the so-called payoff (or objective) function [1,2]. The fact that spring forces depend on displacements, and damping forces on velocities, often entice engineers to design a vibration isolator whose elements, springs and/or dampers, are positioned at the places of putative large relative displacements (or velocities) of the bodies. However, it turns out that for oscillators having several degrees of freedom and modes of vibration, this does not always lead to the optimal solution.

What is said above can at best be illustrated on the practical example of a conventional cars suspension. Because large relative motions between the wheels and the chassis are visible, it seams that a position next to each wheel is the best for springs and dampers to be placed [3–5]. Due to the complexity of the optimization problem many authors used a quarter car model for the optimization purpose (see [6–8] and the references therein). Since in this case the motion of the system is one-dimensional, all springs and dampers act in the direction of motion and their configuration is fixed. Thus, only the spring rates and damper constants can be varied in this optimization. With the goal of maximizing isolation of the chassis from a harmonic base excitation in the frequency domain to achieve the best ride quality of the vehicle, Alkhatib et al. [6] used the root mean square of acceleration or displacement of the chassis as the payoff function. If the interest is in contrary to minimize the dynamic tire load, then the variance of the dynamic load used by Sun et al. [8] serves as the payoff function. The optimization using a half-car model considered for instance by Tamboli and Joshi [9], Giua et al. [10], Sun [11] and a

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full-car model by Jayachandran and Krishnapillai [12] deals again with fixed configurations of springs and dampers while varying their characteristics to meet similar goals. Note, however, that the fixing of special configuration of springs and dampers often exhibits some deficiency in damping of roll vibrations of conventional vehicle suspensions as shown by Le and Pieper [13] in an analysis of forced vibration using a half-car model. The first step in modifying this design concept of suspension by introducing a smart mechanism that adapts the installation ratios of both springs and dampers to different modes of vibrations in equal way has been proposed by Pieper et al. [14]. Nowadays, especially in the tuning of vehicle suspension elements, a huge effort is spend on lap time simulations using different numerical packages [15]. An advanced approach is to measure the real-time motions on a specified system and control it by active springs and dampers. In this case the physical property of each element can be changed immediately and the optimal control is done by software and actuators at each time instant [16–21]. However, this approach only allows an optimization after bad motions have already been detected. The common feature of traditional optimization of passive or active suspensions is that the concept of the dynamic system including the configuration of springs and dampers is fixed at the beginning and only the physical properties of the elements are subject to variation. Independent from the choice of payoff function, this optimization practice limits strongly the variability of dynamical system for comparison to select the overall best solution.

This paper focuses on a new optimization concept for an oscillator with the configuration of springs and dampers being subject to variation. This is realized by a mechanism (rocker) having several motion ratios controlling the action of springs and dampers to each degree of freedom. The variation of motion ratios allows to change the maximum force, induced by springs or dampers, to different modes of vibration. Note that this optimization concept is close to that of topology optimization of materials [22–24] or optimization of placement of piezo-patches in smart structures [25,26]. The springs get used most effectively if the spring energies (and consequently the magnitude of spring forces) are maximal when acting against the corresponding modes of vibration. This leads to the maximum of the total potential action of all springs over one period of each vibration mode. The same can be said in the case of dissipative oscillators with springs and dampers. The aim of this paper is to show that, if the potential action of the springs over one (conditional) period of vibration is used as the payoff function to be maximized among all admissible parameters and motions satisfying Lagrange's equations, then the optimal parameters controlling the action of springs uncouple modes of vibrations and the maximum available forces of springs act against the normal modes.

In order to prove this statement rigorously we need to apply the theory of optimal control processes [27–29] to the special case of time-independent control parameters. In this case we are dealing with the variational problem with constraints imposed on the state variables of the dynamical system in form of the equations of motion depending on the time-independent control parameters. We formulate the extended Pontryagin's maximum principle and, alternatively, the necessary and sufficient conditions for the optimal control parameters of oscillators obeying the equations of small amplitude vibrations. We then apply this theory to the oscillator having two degrees of freedom, first with springs, and then later with springs and dampers, to prove the above statements. For simplicity of the analysis we restrict ourselves to the case of small vibrations for which the springs and dampers can be regarded as linear.

The paper is organized as follows. In the next Section we present the theory of optimal control parameters for oscillators. Sections 3 and 4 apply this theory to the conservative and dissipative oscillators, respectively. Finally, Section 5 discusses the optimal design concept and concludes the paper.

2. Theory of optimal control parameters

We let m-dimensional vector $\mathbf{a} = (a_1, \ldots, a_m)$ denote time-independent control parameters of a mechanical system under consideration and assume that $\mathbf{a} \in \mathcal{A} \subseteq \mathbb{R}^m$, with $\mathcal{A}$ being the set of admissible control parameters. The motion of this mechanical system is governed by the equations

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{a}) \quad (t \geq 0), \\ \mathbf{x}(0) = \mathbf{x}^0, \end{cases} \tag{1}$$

with $\mathbf{f}: \mathbb{R}^n \times \mathcal{A} \to \mathbb{R}^n$. We introduce the payoff function

$$P(\mathbf{a}) = \int_0^T r(\mathbf{x}(t), \mathbf{a})\mathrm{d}t + g(\mathbf{x}(T)), \tag{2}$$

where the end-time $T > 0$, running payoff $r: \mathbb{R}^n \times \mathcal{A} \to \mathbb{R}$ and end-time payoff $g: \mathbb{R}^n \to \mathbb{R}$ are given. The problem is to find optimal parameters $\mathbf{a}^*$ that maximize payoff function (2) among all admissible $\mathbf{a} \in \mathcal{A}$ and $\mathbf{x}(t)$ satisfying constraints (1). Note that the control parameters $\mathbf{a} = (a_1, \ldots, a_m)$ can be identified with the control processes $\mathbf{u}(t) = (u_1(t), \ldots, u_m(t))$ satisfying the constraints

$$\begin{cases} \dot{\mathbf{u}}(t) = \mathbf{0} \quad (t \geq 0), \\ \mathbf{u}(0) = \mathbf{a}. \end{cases}$$

Thus, the above formulated problem is the special case of the problem considered in the theory of optimal control processes [27–29].