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A comparison methodology for measured and predicted displacement fields in modal analysis

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ABSTRACT

Recent advances in experimental mechanics have enabled full-field measurements of deformation fields and - particularly in the field of solid mechanics - methodologies have been proposed for utilizing these fields in the validation of computational models. However, the comparison of modal shapes and the path from the undeformed shape to the deformed shape at the extreme of a vibration cycle is not straightforward. Therefore a new method to compare vibration data from experiment to simulations is presented which uses full-field experimental data from the entire cycle of vibration. Here, the first three modes of vibration of an aerospace panel were compared, covering a frequency range of 14–59 Hz and maximum out-of-plane displacements of 2 mm. Two different comparison methodologies are considered; the first is the use of confidence bands, previously explored for quasi-static loading, the second is the use of a concordance correlation coefficient, which provides quantifiable information about the validity of the simulation. In addition, three different simulation conditions were considered, representing a systematic refinement of the model. It was found that meaningful conclusions can be drawn about the simulation by comparing individual components of deformation from the image decomposition process, such as the relative phase and magnitude. It was ultimately found that the best performing model did not entirely fall within the confidence bounds for all conditions, but returned a concordance correlation coefficient of nearly 70% for all three modes.

1. Introduction

Modal analysis involves establishing the fundamental frequencies and the corresponding deformation shapes of structures during excitation. The analysis can be performed by modeling and/or experiment; and it is good practice to validate the predictions from a model using the measurements from experiments. Validation has been formally defined, by a number of guides [1–3], as establishing the extent to which a model is a reliable representation of the real world. In modal analysis, it is straightforward to compare the resonant frequencies predicted by a model to those measured in experiments. However, the comparison of modal shapes and the path from the undeformed shape to the deformed shape at the extreme of a vibration cycle is less straightforward and becomes more challenging when more complex excitation is present, such as acoustic loading. Recent advances in experimental mechanics have enabled measurements of deformation fields and, in the

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field of solid mechanics, methodologies have been proposed for utilizing these fields in the validation of computational models [4,5]. However, the extension of these methods to include the temporal as well as the spatial domain during cyclic or vibration loading has not been explored and forms the focus of this study.

While the use of full-field methods of experimental strain analysis is gaining momentum in vibration analysis [6], there remain issues associated with the capture of data during fast events which require the use of stroboscopic illumination or expensive high-speed cameras. In addition, it presents new challenges in performing a meaningful comparison with predictions from a simulation due to the large quantity of data and issues with alignment of the coordinate systems, scaling, and interpolation. Therefore, the quantitative comparison of measured and predicted deformation fields for a vibrating component is especially challenging, as it encompasses all of these issues. Hence, the objective of this study was to integrate an elegant technique for data acquisition with an efficient and comprehensive validation methodology to provide an innovative approach to establishing the degree to which the predicted results were a reliable representation of the measured results from an experiment.

Over the past 15 years, a number of publications have addressed the need for verification[1] and validation[2] of computational models. In 1998, the American Institute for Aeronautics and Astronautics (AIAA) published a guide for the computational fluid dynamics community [1]. The Department of Defense has published several iterations of instructions regarding Modeling and Simulation, Verification, Validation, and Accreditation activities, the latest of which was issued in 2009 [2]. In 2005, the Clinical Biomechanics journal issued an editorial statement which defined minimal requirements for a numerical study to be considered for publication [3] and at about the same time the solid mechanics community developed the Guide for Verification and Validation in Computational Solid Mechanics (ASME V&V) [7], which incorporates material published by Oberkampf and his co-workers [8] and provides a framework as well as defining terminology to create a standardized language [9]. Very recently in Europe, a CEN Workshop Agreement has been developed, which provides a protocol for performing a validation process for a computational solid mechanics model [5]; however, the protocol does not address validation of time-varying deformation fields.

Quantitative comparisons between the outcomes from a simulation and experiment should involve a metric that provides an indication of the level of confidence associated with the agreement between the outcomes, taking into account the error and respective uncertainties. An examination of the literature published in the last ten years reveals that a common approach to comparing measured and predicted data is to plot them together on the same graph for a section through the component [10,11]. Typically, a metric is not used when performing this type of comparison and instead, the extent of the agreement is judged qualitatively by how well the two plots match one another. However, to perform a simple quantitative assessment, the relative error between the two sets of data can be calculated by normalizing the difference between the datasets using the measured value [12]. The relative error metric can be effectively applied to point-by-point comparisons and an algebraic sum or mean relative error used for a simple series of data points, but is not very effective or efficient for more complex comparisons. For example, it is not a good choice to compare tensors, or data with time or spatial components; and, the relative error becomes undefined as the measured value approaches zero. Information-rich data fields present similar challenges, and so the comparison is still often reduced to checking a few points rather than using the full data field, e.g. [13]. Recently, the authors proposed a quantitative procedure for comparing fields of deformation, strain or stress data using feature vectors composed of shape descriptors to describe measured and predicted fields [4]. A number of shape descriptors are available including an orthogonal descriptor based on the Zernike polynomials, which was described by Teague [14]. The Zernike descriptor is continuous and valid over a circular domain. An alternative discrete, orthogonal descriptor based on the Tchebichef polynomials was described by Mukundan et al. [15], which unlike the Zernike descriptor, is not invariant to rotation; however, the fact that it is discrete and valid over a rectangular domain makes it well-suited to measured and predicted data fields of strain and displacement. Wang and Mottershead have provided an overview of the different shape descriptors as well as some other methods of image decomposition along with their respective advantages and disadvantages [16]. They noted that the Tchebichef moments are better suited to detecting global features, while Krawtchouk moments are suited to detecting local features [17]. Wang and his colleagues have used a variety of shape descriptors to tackle full-field data from various engineering problems. They used a modified Zernike descriptor to compare the measured strain field around a hole in a plate loaded in tension to the results from a computational model [18]. In addition, they applied the Tchebichef moments to represent mode shapes resulting from vibration measurements during finite element model updating [19], and an adaptive geometric moment descriptor for the vibration analysis of a car hood liner [20]. The adaptive geometric moment descriptor was also used to track the evolution of damage to the car hood liner subjected to an impact from a high-speed projectile [21]. An alternative approach to comparing data fields from modal analysis has been taken by Allemang et al. who used Principal Component Analysis (PCA) to compare two sets of experimental data from frequency testing of an automobile chassis [22].

In this study, the use of a generic decomposition process based on Tchebichef descriptors, which was used previously to compare strain fields in a composite panel subjective to static compressive loading [23], has been extended into the temporal or phase domain for use in the modal analysis of a prototype aircraft panel. In the experiments, pulsed-laser

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[1] verification is defined as “the process of determining that a computational model accurately represents the underlying mathematical model and its solution” [7].

[2] validation is defined as “the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model” [7].