Journal of Sound and Vibration 400 (2017) 369–386

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Analysis of wave motion in one-dimensional structures through fast-Fourier-transform-based wavelet finite element method

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ARTICLE INFO

Article history:
Received 8 June 2016
Received in revised form
10 February 2017
Accepted 15 March 2017
Handling editor: D.J. Wagg
Available online 24 April 2017

Keywords:
B-spine wavelet on the interval
Wavelet finite element
Spectral analysis
Wave modeling

ABSTRACT

This paper presents a hybrid method that combines the B-spline wavelet on the interval (BSWI) finite element method and spectral analysis based on fast Fourier transform (FFT) to study wave propagation in One-Dimensional (1D) structures. BSWI scaling functions are utilized to approximate the theoretical wave solution in the spatial domain and construct a high-accuracy dynamic stiffness matrix. Dynamic reduction on element level is applied to eliminate the interior degrees of freedom of BSWI elements and substantially reduce the size of the system matrix. The dynamic equations of the system are then transformed and solved in the frequency domain through FFT-based spectral analysis which is especially suitable for parallel computation. A comparative analysis of four different finite element methods is conducted to demonstrate the validity and efficiency of the proposed method when utilized in high-frequency wave problems. Other numerical examples are utilized to simulate the influence of crack and delamination on wave propagation in 1D rods and beams. Finally, the errors caused by FFT and their corresponding solutions are presented.

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1. Introduction

Guided wave techniques have been widely applied in structural health monitoring in recent years. The characteristics of wave propagation must be studied to utilize guided wave techniques effectively in damage detection, especially in cases wherein the numerical method is the only method available to solve complex problems. The finite element method (FEM) is the most popular numerical calculation method to analyze the wave propagation. However, conventional FEM may become inaccurate and inefficient when it is used for high-frequency wave problems because of the higher frequency and shorter wavelength of guided waves. It is well known that temporal and spatial discretizations are critical for the accuracy and efficiency in dynamic finite element analysis. A rule of thumb that there should be 10 nodes per wavelength has been widely applied in the design of the finite element mesh [1–4]. Furthermore, Babuška et al. [5,6] found the "pollution error" will lead to that the simple rule of thumb given above is not always adequate and extremely fined meshes are required for large wavenumber. Alternatively, the time step for direct time-integration schemes is usually recommended for one-twentieth of

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http://dx.doi.org/10.1016/j.jsv.2017.03.014
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the smallest period or the ratio of element length to wave speed [7]. These requirements indicate that the size of the system matrix and the number of solving system equations in traditional FEM are extremely large to obtain sufficiently accurate solution, which lead to that the solving process is awfully time-consuming.

Several improved FEMs have been presented to overcome the drawbacks of conventional FEM. Seriani and Oliveira [8] proposed a theoretical analysis of the numerical dispersion of 2D and 3D spectral finite element methods in time-domain for the isotropic elastic wave equation. Chen et al. and Yang et al. [9–11] proposed the BSWI wavelet approach in time domain to solve the wave propagation problems in 1D, arch and 2D structures. Pahlavan et al. [12] applied a wavelet-based spectral finite element approach to linear transient dynamics and elastic wave propagation problems. Joglekar and Mitra [13] proposed a iterative strategy of Fourier spectral finite element method to analyze the interaction of flexural waves with a breathing damage in slender beams. Nguyen et al. [14] modeled the 3D elastic waveguides of arbitrary cross-section embedded in an unbounded solid matrix by combining a semi-analytical finite element method and a perfectly matched layer technique. Kim and Bathe [15] analyzed the numerical dispersion property of the method of finite spheres used with the Bathe method for implicit time integration when it is used for solving transient wave propagation problems.

Wavelet finite element method (WFEM) has elicited much attention because of its high accuracy. The concept of wavelets, which was originally used for signal processing, was introduced by Morlet in 1982 [16]. With the rapid development and application in many areas, wavelet techniques have become the second most significant breakthrough after Fourier transform. In 2001, Dahmen [17] reviewed the developments of wavelet schemes for numerical treatment of partial differential equations, wherein the superiorities of wavelet schemes for numerical calculation were expounded in detail. He Zhengjia and Chen Xuefeng's research group from China focused on the construction method of wavelet basis finite element by using wavelet basis functions as interpolation functions; they successfully applied the technique in structure analysis [18] and the diagnosis of crack faults [19,20]. Wavelets possess three special characteristics, namely, multi-scale, multi-resolution, and compact support [21]. Given their characteristic of multi-scale, wavelets can provide kinds of basis functions as interpolation functions for FEM. The multi-resolution characteristic is highly suitable for the development of adaptive FEM, and wavelet basis functions can localize arbitrary details because of the characteristic of compact support. Since the low-order polynomial as interpolation function is difficult to approximate high-gradient variant fields produced in high-frequency wave problems, compact-support wavelet basis functions will be a better option instead of simple polynomials to construct the finite element. Various wavelets have been used for constructing the wavelet finite element, such as Daubechies [22,23], B-spline [18,19], Hermitian [24,25] and so on. Among of them, the B-spline wavelet on the interval (BSWI) is perhaps the best for numerical calculation of wave propagation [9,21], because of continuity, compact support, and computational efficiency. As mentioned earlier, BSWI FEM has been successfully applied to wave propagation in several simple structures [9–11]; it showed that only a few BSWI elements used can obtain highly accurate results. However, the WFEM must still satisfy the strict requirements of time steps because step-by-step integration in the time domain is still adopted.

Another promising approach is the spectral element method (SEM). Narayanan and Beskos [26] combined the dynamic stiffness method and fast Fourier transform (FFT) and introduced the foundational concept of SEM. Many researchers have developed and applied SEM in the past few decades. Doyle et al. systematically studied the application of SEM to wave propagation in structures, which include not only rods and beams but also vary cross-section waveguide [27], layered solids [28], spectral super-element [29] and so on, and published a monograph in 1997 [30]. Lee et al. [31,32] extensively applied SEM to various problems in structural dynamics. Gopalakrishnan et al. [33,34] focused on the SEM in composites and inhomogeneous media in addition to its application to structural health monitoring and active vibration and wave control. Joglekar and Mitra [13] proposed an iterative strategy extending the SEM to nonlinear analysis. In contrast to the conventional FEM, the spectral element uses exact shape functions and thus treats the mass distribution within the structural element exactly [34]. Accordingly, a regular structural member of any length but without discontinuities in geometry or material properties can be modeled with a single spectral element [31]. However, obtaining an exact wave solution is not always possible, especially when dealing with highly complex and multi-dimensional problems. Thus, the application of the classical SEM has been limited to some extent.

In this study, a hybrid method that combines WFEM and FFT-based spectral analysis is developed to overcame the disadvantage of WFEM in temporal resolution and extend the modeling flexibility of SEM. Wherein, the compacted-support BSWI scaling functions are utilized to approximate the theoretical wave solution in the spatial domain and construct a high-accuracy dynamic stiffness matrix. To verify the effectiveness of the proposed method, a comparative analysis of four different methods (conventional FEM, WFEM, SEM, and the proposed method) is conducted, and the spatial and temporal resolution requirements of the four different methods are discussed in detail. Moreover, several damage models are introduced to determine the effect of crack and delamination on wave propagation in rod and beam. Finally, the errors and their corresponding solutions are discussed.

2. FFT-based BSWI approach in the frequency domain

2.1. Spatial approximation of wave solution through BSWI scaling function

As mentioned in introduction, the compacted-support BSWI defined in $L^2\left[0,1\right]$ is very suitable to approximate the theoretical wave solution in the spatial domain. For scale j and the mth order BSWI (simply denoted as BSWIm_j hereafter)

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