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Non-stationary random vibration analysis of structures under multiple correlated normal random excitations

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ABSTRACT

An algorithm that integrates Karhunen-Loeve expansion (KLE) and the finite element method (FEM) is proposed to perform non-stationary random vibration analysis of structures under excitations, represented by multiple random processes that are correlated in both time and spatial domains. In KLE, the auto-covariance functions of random excitations are discretized using orthogonal basis functions. The KLE for multiple correlated random excitations relies on expansions in terms of correlated sets of random variables reflecting the cross-covariance of the random processes. During the response calculations, the eigenfunctions of KLE used to represent excitations are applied as forcing functions to the structure. The proposed algorithm is applied to a 2DOF system, a 2D cantilever beam and a 3D aircraft wing under both stationary and non-stationary correlated random excitations. Two methods are adopted to obtain the structural responses: a) the modal method and b) the direct method. Both the methods provide the statistics of the dynamic response with sufficient accuracy. The structural responses under the same type of correlated random excitations are bounded by the response obtained by perfectly correlated and uncorrelated random excitations. The structural response increases with a decrease in the correlation length and with an increase in the correlation magnitude. The proposed methodology can be applied for the analysis of any complex structure under any type of random excitation.

1. Introduction

Many engineering structures are subjected to random dynamic loads which might be stationary or non-stationary, uncorrelated or correlated in time and/or in space. Random excitations, which lack deterministic definition in time and/or space domains, often occur in many practical vibration problems, e.g. gust loads on aircraft wings, excitations caused by turbulent boundary layers on panels, and non-stationary wind and seismic loads on tall buildings. For the convenience of the random analysis under non-stationary and correlated random excitations, such loads are often simplified to be stationary and uncorrelated. However, the development of the treatment of non-stationary correlated excitations is inhibited

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by both the extensive computational cost and the inherent mathematical intricacies. With the relentless progress in high-performance computing, computational methods [1] are being increasingly used to perform random vibration analyses of complex structures. Hence, non-stationary random vibration analysis has been drawing increasing attention in recent years from the designers of many civil, mechanical and aerospace structures.

Stationary random excitations can be described in time or frequency domain respectively using the autocorrelation function and power spectral density (PSD) function. Since it is easier to solve random vibration problem in the frequency domain, PSD functions are usually adopted to represent stationary random excitations and responses [2–4]. For a non-stationary random process, the marginal probability density function (PDF) of excitation may only have positive real domains that might be bounded. So, the covariance function is used to define the random processes in the time domain. The structural response under non-stationary random excitations [5–7] has been conducted for simple systems using analytical or numerical methods for the response calculations. However, the structural response analysis under non-stationary and correlated random excitations, which are functions of both time and space, has not been studied in great depth. For the most part, such excitations are decomposed into a stationary random process and a modulating function [8].

When calculating the structural response under non-stationary random excitation, the first step is to discretize the auto-covariance functions of excitations [9]. Recently, efforts have been made to obtain the response using Karhunen-Loeve expansion (KLE), Polynomial Chaos expansion [10], and Monte-Carlo method [11]. The KLE is a useful and efficient tool for discretizing second-order random processes with known covariance function [12,13] by solving the Fredholm integral equations. Recently, an algorithm for decomposing the auto-covariance function in KLE using orthogonal decomposition has been presented by Mulani et al. [5,14] and Phoon et al. [15]. In this method, the eigenvalues and eigenfunctions are obtained by using the Galerkin projection method [16] and the collocation technique [17]. The Galerkin projection method is an efficient method for solving eigenvalues and eigenfunctions of auto-covariance. However, it often yields negative eigenvalues of the auto-covariance function (based upon the type of basis functions). It is critical to select suitable basis functions for the KLE during Galerkin projection method. In the authors' previous work, different basis functions were chosen to study random vibration, namely: global trigonometric basis functions which require the user to have some *a priori* knowledge of the system response [5], and piecewise linear or higher order polynomial Lagrange interpolation functions [18,19] which neither yield negative nor infinite-valued eigenvalues for the auto-covariance function. They also do not yield any large inaccuracies. The collocation technique is another efficient method which is used to obtain the eigenvalues and eigenvectors of the auto-covariance matrix by converting the auto-covariance function into an auto-covariance matrix. The collocation technique is not as accurate as the Galerkin projection method. However, this method does not yield negative and/or infinite-valued eigenvalues [5].

If the random processes are uncorrelated, then the KLE can be applied to each of the processes that can be constructed separately [20]. However, if the random processes are correlated, then the random processes can't be expanded into consistent expansions in a straightforward manner. Nevertheless, many engineering systems involve multiple correlated random processes. Thus, it is necessary to develop appropriate methods to model and simulate multiple correlated random processes in both time and spatial domains effectively. The spectral density method [21–23] has already been extended to multivariate random processes. However, the spectral density method is limited to stationary random processes, and it can only consider the spatial correlation of random processes. Another technique that has been proposed to represent multiple correlated processes is the probabilistic principal component analysis [24,25]. In this method, multiple random processes are represented in terms of a linear combination of independent random variables. This method assumes that all of the random variables representing a process are independent, and therefore, the method cannot properly represent cross-correlated process [26,27]. An algorithm which relies on expansions in terms of correlated sets of random variables reflecting the cross-covariance structure of the processes has been proposed [27].

In previous work [18], the non-stationary random vibration analysis of systems under uncorrelated random excitation was carried out, and it has shown its applicability, efficiency, and accuracy in solving simple and complex systems. However, the method as developed by the authors was still not developed for correlated random excitations. To obtain a method that is suitable for random vibration analysis under multiple random excitations that are correlated in both time and space, and to reduce the computational cost of generating response statistics, an algorithm that integrates KLE and FEM is proposed to conduct the random vibration analysis for any dynamic system, simple or complex; and for any random excitation, stationary or non-stationary. The proposed algorithm has been applied to a simple 2DOF system, a cantilever beam and an aircraft wing subjected to both stationary and non-stationary excitations.

2. Theoretical derivation

2.1. Karhunen-Loeve expansion for multiple correlated excitation

A time-invariant second-order system subjected to a forcing function can be written as:

$$\mathbf{M}\ddot{x} + \mathbf{C}\dot{x} + \mathbf{K}x + \mathbf{H}(x)x = \mathbf{f}(t, \omega) \tag{1}$$

where $\mathbf{M}$ and $\mathbf{C}$ and are mass, damping matrix of the system, $\mathbf{K}$ is the linear elastic stiffness matrix, $\mathbf{H}(x)$ is the generalized