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Response statistics of rotating shaft with non-linear elastic restoring forces by path integration

Oleg Gaidai [a,*], Arvid Naess [b], Michael Dimentberg [c]

[a] Jiangsu University of Science and Technology, Zhenjiang, China
[b] Norwegian University of Science and Technology, Trondheim, Norway
[c] Worcester Polytechnic Institute, Worcester, USA

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A B S T R A C T

Extreme statistics of random vibrations is studied for a Jeffcott rotor under uniaxial white noise excitation. Restoring force is modelled as elastic non-linear; comparison is done with linearized restoring force to see the force non-linearity effect on the response statistics. While for the linear model analytical solutions and stability conditions are available, it is not generally the case for non-linear system except for some special cases. The statistics of non-linear case is studied by applying path integration (PI) method, which is based on the Markov property of the coupled dynamic system. The Jeffcott rotor response statistics can be obtained by solving the Fokker–Planck (FP) equation of the 4D dynamic system.

An efficient implementation of PI algorithm is applied, namely fast Fourier transform (FFT) is used to simulate dynamic system additive noise. The latter allows significantly reduce computational time, compared to the classical PI.

Excitation is modelled as Gaussian white noise, however any kind distributed white noise can be implemented with the same PI technique. Also multidirectional Markov noise can be modelled with PI in the same way as unidirectional.

PI is accelerated by using Monte Carlo (MC) estimated joint probability density function (PDF) as initial input. Symmetry of dynamic system was utilized to afford higher mesh resolution. Both internal (rotating) and external damping are included in mechanical model of the rotor.

The main advantage of using PI rather than MC is that PI offers high accuracy in the probability distribution tail. The latter is of critical importance for e.g. extreme value statistics, system reliability, and first passage probability.

1. Introduction

The response of rotating shaft to a vertical random excitation is of practical concern for road vehicles with external force entering system through the shaft supports. Exciting force can be modelled as uniaxial vertical one, having white noise nature, due to the road irregularities [5]. Even for transport engineering rotating machinery with excitation force acting in two different transverse directions, it is still of interest to consider first the case of uniaxial excitation, see [1].

* Corresponding author.
E-mail address: oleg.gaidai@yahoo.com (O. Gaidai).

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It should be noted however that presented in this paper path integration (PI) methodology, enhanced with Fast Fourier Transform (FFT) equally applies to multiaxial excitation force, as long as Markov condition of independent force increments holds. The latter makes PI relevant for wide range of Jeffcott type rotating shaft applications: not only in transport but also in fluid machinery, e.g. turbo-pump for a liquid-propellant rocket engine [6,12].

Since the road irregularity in automotive design is often modelled using normally distributed random process [5], it is of interest to study Gaussian white noise excitation force.

For the linear system with linearized restoring force, stability conditions are well studied [1,7]. This paper shows that by including the stiffening non-linearity of restoring force, stable steady-state whirl response may be obtained beyond linear instability margin. The latter is of practical importance, since the actual restoring spring is non-linear.

The PI method is an efficient approximation for solving the Fokker–Planck (FP) equation and providing the stationary or nonstationary response probability density functions (PDF) of the dynamic system. This method is based on the Markov property of the dynamic system and the evolution of the response PDF is calculated via a step-by-step solution technique based on short time steps. Specifically, the response PDF at a given time instant can be obtained when the response PDF at an earlier close time instant as well as the conditional PDF (which is of Gaussian form) are already known.

This paper studies random vibrations of a Jeffcott rotor under uniaxial white noise excitation. Restoring force is modelled as elastic non-linear since it may be in fact the case in engineering. The statistics of non-linear case is studied by applying 4D path integration (PI) method, with main focus on obtaining high resolution PDF tail. An advantage of PI compared to direct Monte Carlo (MC) simulation is shown. The latter makes PI of practical engineering interest when doing reliability analysis of rotating shafts of Jeffcott type.

2. Analysis

Numerous studies have been devoted to non-linear effects in rotor dynamics [8,9,11–15]. This paper studies a simple Jeffcott rotor with disk of mass m and non-linear restoring force. The linearized axisymmetric lateral stiffness of the combined shaft/support is denoted K, and the shaft angular velocity is denoted ν. The shaft external (non-rotating) damping and internal (rotating) damping coefficients are c_n and c_r respectively.

The shaft Z-axis is assumed to be horizontal and exciting force is assumed to act in vertical X-direction. Let $X(t)$ and $Y(t)$ be the disk's mass transverse non-dimensionalized displacements along the force (excited, vertical)X-axis and along perpendicular (non-excited)Y- axis, respectively.

Then, neglecting the gravity force for sufficiently high rotation speed and adding the lateral vertical random force in Xdirection, one obtains following equations of the disk lateral motion [5]

$$\ddot{X}+2\kappa\dot{X}+f_X(X,Y)+2\beta\nu Y=\gamma\dot{W}(t);\ \ddot{Y}+2\kappa\dot{Y}+f_Y(X,Y)-2\beta\nu X=0 \tag{1}$$

with $\kappa=\alpha+\beta$, $\alpha=c_n/2m$, $\beta=c_r/2m$. $\dot{W}(t)$ is a stationary zero-mean Gaussian white noise, being derivative of the Wiener process $W(t)$ with normally distributed incitements $E[dW(t)]=0$ and $E[dW(t)dW(t+dt)]=\delta(dt)$. Rotation speed is denoted by ν, noise intensity by γ. Note that the dynamic system (1) is asymmetric (uniaxial) exciting force, because noise acts only along transverse vertical Xdirection. For the study of symmetric (biaxial) Jeffcott system, similar to (1), see [12] where the analytical solution to the FP equation had been obtained.

The non-linear elastic restoring forces along X and Y axes are denoted $f_X(X,Y)$ and $f_Y(X,Y)$ respectively. In case restoring forces are linearized, one has

$$f_X(X,Y)=\Omega^2X;\quad f_Y(X,Y)=\Omega^2Y \tag{2}$$

with $\Omega^2=K/m$, see [1]. The non-linear version of restoring forces is assumed as in [5]

$$f_X(X,Y)=\frac{\Omega^2X}{1+r(X,Y)}\left[1+\varepsilon^{-1}r(X,Y)\right];\quad f_Y(X,Y)=\frac{\Omega^2Y}{1+r(X,Y)}\left[1+\varepsilon^{-1}r(X,Y)\right] \tag{3}$$

with $r(X,Y)=\sqrt{1+X^2+Y^2}-1$, $\varepsilon=T/EA$ with T being the reference tension in the shaft, corresponding to its horizontal position, i.e. $T=EA\varepsilon$. For nonlinear system the ε-terms in Eq. (3) have been taken into account, while for linearized system ε is set to 0.

It should be noted that in this paper the shaft is assumed to be perfectly balanced. As mentioned above, X and Y are non-dimensionalized displacements, related to actual transverse displacements $u$and v by scaling $X=u/L, Y=v/L$ with Lbeing half distance between shaft supports [5]. Since disk's mass transverse displacements are small compared to the shaft length, $X, Y \ll 1$ and therefore $r(X,Y)=O(X^2+Y^2)$ is of the second order, justifying linearization in Eq. (2) if the non-dimensionalized displacements $X.\ Y=o(1)$are small.

Since the main focus of this paper is extreme response statistics, linearization is not proper. Therefore efficient PI