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Parametric instability of spinning elastic rings excited by fluctuating space-fixed stiffnesses

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ABSTRACT

This study investigates the vibration of rotating elastic rings that are dynamically excited by an arbitrary number of space-fixed discrete stiffnesses with periodically fluctuating stiffnesses. The rotating, elastic ring is modeled using thin-ring theory with radial and tangential deformations. Primary and combination instability regions are determined in closed-form using the method of multiple scales. The ratio of peak-to-peak fluctuation to average discrete stiffness is used as the perturbation parameter, so the resulting perturbation analysis is not limited to small mean values of discrete stiffnesses. The natural frequencies and vibration modes are determined by discretizing the governing equations using Galerkin's method. Results are demonstrated for compliant gear applications. The perturbation results are validated by direct numerical integration of the equations of motion and Floquet theory. The bandwidths of the instability regions correlate with the fractional strain energy stored in the discrete stiffnesses. For rings with multiple discrete stiffnesses, the phase differences between them can eliminate large amplitude response under certain conditions.

1. Introduction

Gears used in aerospace applications are designed to be thin to reduce weight. These thin, compliant gears experience large loads and operate at high rotation speeds, so elastic gear deformation is a substantial issue. Gear elastic vibrations are experimentally found in Refs. [1,2]. Excitation in geared systems comes from the changing contact conditions on the gear teeth as the gears rotate kinematically. In many dynamic models, this excitation is represented by fluctuating mesh stiffnesses. When the fluctuation frequency is near resonant gear speeds large dynamic response occurs, which leads to large dynamic loads, potential structural failure, and noise.

Gear weight reduction is achieved by using thin webs with large facewidths (for the large loads) or ring-like gears with no webs. Such lightweight gears are prone to deform elastically, and can be modeled geometrically as rings. The vibration of spinning, elastic rings has been intensively investigated in literature. Carrier [3] derived the governing equations of a rotating ring with in-plane flexural vibrations. General equations for the vibration of rotating bodies and the natural frequencies of a rotating ring were obtained by Johnson [4]. Bert and Chen [5] investigated the bending and twisting vibrations of rotating rings on a uniform elastic foundation. Their analysis included in-plane and out-of-plane deformations. Using

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Hamilton's principle, Bickford and Reddy [6] derived the equations of motion for rotating rings with in-plane vibrations. The governing equations for rotating free rings were derived in the stationary reference frame using Lagrangian strain by Kim and Chung [7]. Rao and Sundararajan [8] studied the vibration of nonrotating rings with fixed supports. Allaei et al. [9] investigated the vibration of nonrotating rings attached to lumped masses and stiffnesses. Wu and Parker [10] studied the vibration of nonrotating rings on a general elastic foundation. Vangipuram Canchi and Parker [11] studied the parametric instability of rotating rings subjected to moving, time-varying springs. Their equations of motion were derived in the rotating, ring-fixed reference frame. Cooley and Parker [12] investigated the vibration of rotating elastic rings coupled to space-fixed discrete stiffnesses. Cooley and Parker [13] showed that an inextensible model does not accurately predict the vibrations of elastic rings that rotate at high speeds or are attached to discrete stiffnesses. The response of this system to periodic fluctuations of the discrete stiffnesses, which is practically relevant for the dynamic response in gear systems, has not been addressed.

Investigation into parametric instability of lumped-parameter gear models can be found in Refs. [14–19], for example. Fewer works have investigated the parametric instability of compliant gears. Vangipuram Canchi and Parker [20] studied the parametric instability of nonrotating rings subjected to moving, time-varying springs. This work was extended to rotating rings in Ref. [11]. In these works the average spring stiffnesses are assumed to be small compared to ring bending stiffness, and they mainly focus on the instabilities associated with the rotation of the springs. Parker and Wu [21] investigated the parametric instability of planetary gears with stationary, elastic ring gears. For geared systems parametric excitation causes large amplitude vibration and instability. In other systems, parametric excitation can reduce vibrations [22–24].

This study investigates the parametric instability of spinning rings excited by an arbitrary number of periodically fluctuating discrete stiffnesses. The instability boundaries are derived in closed-form using the method of multiple scales [25]. This paper starts from the vibration model of a rotating ring proposed by Cooley and Parker [12], where the discrete stiffnesses coupled with the rotating ring are included in the eigenvalue problem. The perturbation analysis is formulated using the ratio of peak-to-peak fluctuation to the average discrete stiffnesses as the small parameter, so the analysis is not limited to small average values of discrete stiffnesses. Parametric instability regions are compared to a similar model that assumes small average discrete stiffnesses proposed by Vangipuram Canchi and Parker [11]. The effects of the phase differences between time-varying discrete stiffnesses on the bandwidths of the instability regions are investigated for rings with two stiffnesses and with multiple equally spaced stiffnesses. The accuracy of the perturbation results is investigated by comparing them to results obtained by direct numerical integrations and Floquet theory. Numerical results are demonstrated for compliant gears used in aircraft engine applications.

2. Analytical model

The uniform elastic ring shown in Fig. 1 rotates at constant speed Ω. It vibrates elastically with radial ($u(\theta, t)$) and tangential deformations ($v(\theta, t)$). To accurately model the vibrations of these high-speed spinning structures connected to discrete stiffnesses, no inextensibility constraints are employed [13]. The ring's neutral axis radius is R, cross-sectional area is A, and cross-sectional area moment of inertia relative to the neutral axis is I. The ring has density ρ and elastic modulus E. The ring is supported by an elastic foundation with radial ($\tilde{k}_r$) and tangential ($\tilde{k}_\theta$) stiffness per unit arclength. The subsequent formulation uses the elastic foundation stiffnesses $k_r = R\tilde{k}_r$ and $k_\theta = R\tilde{k}_\theta$, where k_r and k_θ are the elastic foundation stiffnesses per radian of angular variation. The ring is attached to N_s space-fixed, tangentially oriented, periodically fluctuating (with period $2\pi/\Omega_m$, where Ω_m is the fluctuation frequency) discrete stiffnesses k_{mi} located circumferentially around the ring

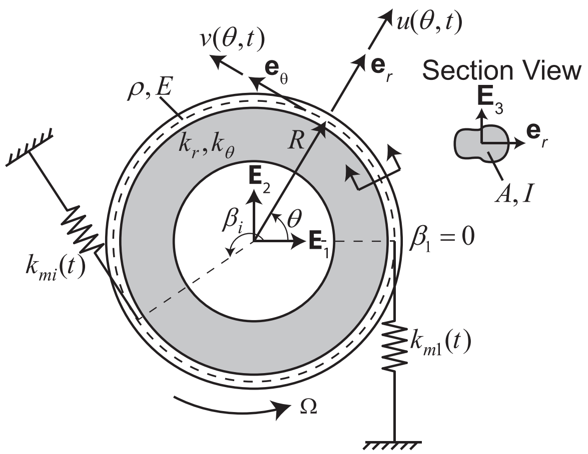

Fig. 1. Schematic of the rotating, elastic ring model with an arbitrary number of space-fixed, fluctuating discrete stiffnesses. The $\{\mathbf{E}_1, \mathbf{E}_1\}$ basis is fixed in space. The radial (u) and tangential (v) deformations are measured with respect to the stationary, cylindrical $\{\mathbf{e}_r, \mathbf{e}_\theta\}$ basis.