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Identification of the flexural stiffness parameters of an orthotropic plate from the local dynamic equilibrium without a priori knowledge of the principal directions

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ABSTRACT

This paper proposes an inverse method to characterize orthotropic material properties from vibratory measurements on plate-like structures. The method is an adaptation of the Force Analysis Technique (FAT), which was originally developed to identify the external force distribution acting on a structure using its local discretized equation of motion. This method was recently adapted to the identification of elastic and damping properties of isotropic plates. In the present approach, the equation of motion of an orthotropic plate with respect to an arbitrary set of orthogonal axes is considered. The angle between the axes of the measurement mesh and the principal directions of orthotropy therefore explicitly appears in the equation and constitutes an unknown. A procedure to identify this angle together with the flexural stiffness parameters is proposed, as well as an automatic regularization procedure to overcome the high sensitivity of the inverse problem to measurement noise. The method is illustrated using simulated data. Experimental results shown on various structures demonstrate the ability of the method to simultaneously identify the principal orthotropy directions and the flexural stiffness parameters.

1. Introduction

Composite materials are increasingly used in the design of structural parts, due to their better strength-to-weight ratio compared to metal. This characteristic allows to design lightweight structures, mostly for energy saving reasons. Moreover, composite materials also enable more flexibility in structural engineering. For instance in the case of laminates, the different fibers and matrix materials available and the choice of the stacking sequence (number and orientation of plies) offer a myriad of possible combinations. This gives the potential to design materials having target mechanical properties depending on requirements of a specific application. However, none of the various existing manufacturing processes can absolutely ensure that the properties of the produced material will perfectly match those theoretically designed. It is therefore often necessary to characterize the material at the end of the process.

Most composite panels used in the industry exhibit an orthotropic behavior, which is fully determined by 4 independent material properties: two Young's moduli, one shear modulus and one Poisson's ratio. Several experimental methods exist to

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estimate them once the composite is made. Static testing is very common, since standard methods are available [1]. However, they provide limited information about the elastic properties, which are often frequency-dependent in the case composite materials. To address this issue, vibration-based methods are also used. They generally rely on experimental modal analysis [2], in the sense that the elastic parameters are deduced from the measurement of resonance frequencies and possibly mode shapes. The most common is the Oberst technique, which involves the vibration of a clamped-free beam. Some methods are particularly suited to orthotropic materials and enable the identification of several parameters using the same sample plate [3].

The main limitation of usual modal methods is that they are not valid at medium and high frequencies, i.e. in frequency domains with high modal overlap. To enable the identification of modal parameters up to the mid-frequency domain, high resolution methods have been developed [4]. They were successfully applied to the measurement of elastic and damping properties of composite plates [5]. However, another limitation of modal methods is that they require specific boundary conditions, since modal parameters are global properties of the structure.

Alternatively, wavenumber fitting approaches have been used to estimate material properties [6–10]. In particular, the Inhomogeneous Wave Correlation technique (IWC) [7,8] is particularly well suited to orthotropic plates. It is based on the projection of a local displacement field onto damped plane waves to identify the natural wavenumber of the structure, for various angles of propagation. In a second step, the theoretical dispersion relation is used to identify flexural stiffness parameters from these wavenumber estimates. The method is insensitive to boundary conditions and allows the identification of material properties at arbitrary frequencies, offering advantages over modal approaches. However, since the method assumes that a wave exists in each direction, special care must be taken to reject spurious wavenumber estimates, especially in a frequency range where the modal overlap is low. This reduces the amount of information available to perform the second step, which may alter the quality of the identification of material properties.

Recently, another approach has emerged to identify material properties from a local vibration field. It is based on the principle that the measured displacements must verify the local equation of motion of the structure. This way of formulating the inverse problem was inspired from the Force Analysis Technique (FAT) [11,12], a method to identify vibration sources. It uses a finite difference scheme to approximate spatial derivatives in the equation of motion and a regularization step (windowing and filtering) to overcome the high sensitivity to measurement uncertainties. The identification of material properties using this approach has been first presented in [13] in the case of isotropic plates. Recent extensions of the method concern the use of a corrected finite difference scheme to avoid the regularization step [14] and the simultaneous identification of Young's and shear modulus on a beam, using Timoshenko's equation of motion [15]. The method presented in Ref. [13] could be straightforwardly extended to the orthotropic case simply by changing the considered equation of motion into that of an orthotropic plate. However, this would require the measurement mesh to be aligned with the principal directions of the material. There are cases, however, where the principal directions are either not known a priori, or could slightly deviate from their nominal orientation due to the making process.

The aim of this paper is to propose a technique to simultaneously identify the stiffness parameters and the orthotropy angle from the single measurement of a local displacement field, based on the verification of the local equation of motion. The proposed technique does not require a priori knowledge of the orthotropy directions, nor that the measurement mesh be aligned with them.

In the second section of this paper, the principle of the identification technique is developed. In the third section, the method is illustrated using simulated data. An automatic regularization procedure is proposed to overcome the high sensitivity to measurement noise. The last section shows experimental results obtained on various orthotropic structures.

2. Theory

2.1. Equation of motion

The equation of motion of a thin orthotropic plate in harmonic regime is

$$D_1\frac{\partial^4 w}{\partial x^4} + (D_2 + D_4)\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_3\frac{\partial^4 w}{\partial y^4} - \rho h \omega^2 w(x, y) = f(x, y), \tag{1}$$

where x and y correspond to the orthogonal principal directions of the orthotropic material, that are neither necessarily collinear with the edges of the plate (in the case of rectangular geometry) nor with the axes of a measurement mesh, $w(x, y)$ is the transverse displacement, $f(x, y)$ the distribution of external forces, ω the angular frequency of excitation, ρ the density, h the thickness of the plate and D_1, D_2, D_3, D_4 four independent flexural stiffness parameters. The latter are related to Young's moduli E_x and E_y, shear modulus G_{xy}, and Poisson's ratios ν_{xy} and ν_{yx} of the material,

$$D_1 = \frac{E_x h^3}{12(1 - \nu_{xy}\nu_{yx})}, \tag{2a}$$

$$D_2 = \frac{E_x \nu_{yx} h^3}{6(1 - \nu_{xy}\nu_{yx})} = \frac{E_y \nu_{xy} h^3}{6(1 - \nu_{xy}\nu_{yx})}, \tag{2b}$$