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Random vibration of linear and nonlinear structural systems with singular matrices: A frequency domain approach

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ABSTRACT

A frequency domain methodology is developed for stochastic response determination of multi-degree-of-freedom (MDOF) linear and nonlinear structural systems with singular matrices. This system modeling can arise when a greater than the minimum number of coordinates/DOFs is utilized, and can be advantageous, for instance, in cases of complex multibody systems where the explicit formulation of the equations of motion can be a nontrivial task. In such cases, the introduction of additional/redundant DOFs can facilitate the formulation of the equations of motion in a less labor intensive manner. Specifically, relying on the generalized matrix inverse theory, a Moore-Penrose (M-P) based frequency response function (FRF) is determined for a linear structural system with singular matrices. Next, relying on the M-P FRF a spectral input-output (excitation-response) relationship is derived in the frequency domain for determining the linear system response power spectrum. Further, the above methodology is extended via statistical linearization to account for nonlinear systems. This leads to an iterative determination of the system response mean vector and covariance matrix. Furthermore, to account for singular matrices, the generalization of a widely utilized formula that facilitates the application of statistical linearization is proved as well. The formula relates to the expectation of the derivatives of the system nonlinear function and is based on a Gaussian response assumption. Several linear and nonlinear MDOF structural systems with singular matrices are considered as numerical examples for demonstrating the validity and applicability of the developed frequency domain methodology.

1. Introduction

The dynamic analysis of systems subjected to stochastic excitations has been extensively studied over the last decades; see for instance Refs. [1,2] and [3] for some indicative books, as well as Refs. [4] and [5] for some recent techniques related to the path integral concept. In general, in the field of random vibration of structural systems [6] modeling the system by utilizing the minimum number of coordinates (generalized coordinates) yields not only non-singular, but also positive

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definite matrices. Note, however, that an alternative modeling of the system equations of motion that employs additional/redundant degrees-of-freedom (DOFs)/coordinates may be preferable, particularly in the field of multi-body system dynamics, for a number of reasons. These may include decreased complexity and computational cost associated with the formulation of the equations of motion. Thus, although formulating the equations of motion by employing redundant DOFs yields singular system matrices, this alternative modeling scheme appears advantageous in many cases; see Refs. [7–15] for a detailed discussion on the topic.

Clearly, determining the dynamic response of structural systems with singular matrices poses significant challenges as standard solution techniques such as those based on a state-variable formulation cannot be utilized, at least in a straightforward manner. In this regard, relying on the concept of the Moore-Penrose (M-P) generalized inverse Udwadia and co-workers (e.g. [16]) determined the dynamic response of systems with singular matrices subject to deterministic excitation. Subsequently, the authors developed in Refs. [17,18] generalized random vibration time-domain techniques for determining the response of linear and nonlinear structural systems subject to stochastic excitations; see also Ref. [19] for an alternative treatment based on polynomial matrix theory.

In this paper, standard frequency domain random vibration solution methodologies (e.g. [6]) are generalized to account for systems with singular matrices. To this aim, based on the theory of generalized matrix inverses, an M-P based frequency response function (FRF) is derived for a structural system with singular matrices. Next, relying on the M-P FRF the celebrated standard input-output (excitation-response) relationship in the frequency domain is generalized for determining the system response power spectrum. Finally, the above derived frequency domain relationship is utilized in conjunction with a recently developed statistical linearization technique [18] for determining the response statistics of nonlinear systems with singular matrices. The validity of the herein developed frequency domain random vibration techniques is demonstrated by pertinent numerical examples including several linear and nonlinear systems with singular matrices.

2. Moore-Penrose theory elements

In this section, some elements of the generalized matrix inverse theory pertaining to the Moore-Penrose (M-P) matrix inverse, are provided for completeness.

Definition 1. If $\mathbf{A} \in \mathbb{C}^{m \times n}$ then $\mathbf{A}^+$ is the unique matrix in $\mathbb{C}^{n \times m}$ so that

$$\begin{aligned} &\mathbf{A}\mathbf{A}^+\mathbf{A} = \mathbf{A}, \quad \mathbf{A}^+\mathbf{A}\mathbf{A}^+ = \mathbf{A}^+, \\ &(\mathbf{A}\mathbf{A}^+)^* = \mathbf{A}\mathbf{A}^+, \quad (\mathbf{A}^+\mathbf{A})^* = \mathbf{A}^+\mathbf{A}. \end{aligned} \tag{1}$$

The matrix $\mathbf{A}^+$ is known as the M-P inverse of $\mathbf{A}$ and Eq. (1) represents the so-called M-P equations. In general, the M-P inverse of a square matrix exists for any arbitrary $\mathbf{A} \in \mathbb{C}^{n \times n}$, and if $\mathbf{A}$ is non-singular, $\mathbf{A}^+$ coincides with $\mathbf{A}^{-1}$. Further, the M-P inverse of any $m \times n$ matrix $\mathbf{A}$ can be determined, for instance, via a number of recursive formulae (e.g., [20,21]), and provides a tool for solving equations of the form

$$\mathbf{A}\mathbf{x} = \mathbf{b}, \tag{2}$$

where $\mathbf{A}$ is a rectangular $m \times n$ matrix, $\mathbf{x}$ is an n vector and $\mathbf{b}$ is an m vector. For a singular square matrix $\mathbf{A}$, i.e. $det\mathbf{A} = 0$, utilizing the M-P inverse, Eq. (2) yields

$$\mathbf{x} = \mathbf{A}^+\mathbf{b} + (\mathbf{I} - \mathbf{A}^+\mathbf{A})\mathbf{y}, \tag{3}$$

where $\mathbf{y}$ is an arbitrary n vector and $\mathbf{I}$ is the identity matrix. A more detailed presentation of the topic can be found in Refs. [20] and [22].

3. Frequency domain stochastic response analysis of linear systems with singular matrices

In this section, the response of linear systems with singular matrices subject to stochastic excitation is determined via a frequency domain approach. Note that the herein developed frequency domain response analysis methodology can be construed as an alternative to a recently developed time domain technique [17].

3.1. Linear systems with standard non-singular matrices

Some elements of the frequency domain stochastic response analysis of systems with standard non-singular matrices are provided in the following for completeness. In this regard, the statistics of the system response, $\mathbf{q}(t)$, to an external excitation, $\mathbf{Q}(t)$, are determined in the frequency domain by utilizing input-output relationships, involving the FRF matrix $\boldsymbol{\alpha}(\omega)$ [6]. Specifically, consider the equations of motion of an n-DOF linear system given by