Journal of Sound and Vibration 409 (2017) 65–80

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A highly precise frequency-based method for estimating the tension of an inclined cable with unknown boundary conditions

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ARTICLE INFO

Article history:
Received 8 October 2016
Received in revised form
20 July 2017
Accepted 22 July 2017
Handling Editor: L.G. Tham

Keywords:
Cable-stayed bridge
Frequency-based method
Finite difference method
Eigenvalue derivatives

ABSTRACT

This paper develops a method for precisely determining the tension of an inclined cable with unknown boundary conditions. First, the nonlinear motion equation of an inclined cable is derived, and a numerical model of the motion of the cable is proposed using the finite difference method. The proposed numerical model includes the sag-extensibility, flexural stiffness, inclination angle and rotational stiffness at two ends of the cable. Second, the influence of the dynamic parameters of the cable on its frequencies is discussed in detail, and a method for precisely determining the tension of an inclined cable is proposed based on the derivatives of the eigenvalues of the matrices. Finally, a multi-parameter identification method is developed that can simultaneously identify multiple parameters, including the rotational stiffness at two ends. This scheme is applicable to inclined cables with varying sag, varying flexural stiffness and unknown boundary conditions. Numerical examples indicate that the method provides good precision. Because the parameters of cables other than tension (e.g., the flexural stiffness and rotational stiffness at the ends) are not accurately known in practical engineering, the multi-parameter identification method could further improve the accuracy of cable tension measurements.

1. Introduction

Inclined cables are one of the main support components of cable-stayed bridges. The cable tension is closely correlated with the internal forces and deflection of the bridge. Therefore, accurately determining the cable tension is directly related to the security of the bridge during both construction and service. Several methods can be used to measure cable tension, including the hydraulic pressure meter method, tension/compression load cell method, vibration method [1–10] and electromagnetic method [11]. The vibration method is widely used during both the construction and service stages because of its simplicity and efficiency. This method is based on the principle that the cable stiffness is mainly provided by its tension and that a quantitative relationship exists between the natural frequency of a cable and its tension. Therefore, the cable tension can be obtained by measuring the natural vibration frequencies of the cable. However, practice has shown that estimating the cable tension using vibration methods results in non-negligible errors, mainly because the flexural stiffness, sag effect, boundary conditions and inclination angle of the cable are not properly considered.

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http://dx.doi.org/10.1016/j.jsv.2017.07.043

In 1974, Irvin and Caughey [1] studied cable vibration and presented an analytical solution for the equation of nonlinear free cable vibration. Irvin [2] wrote a monograph about cable vibration theory in the early 1980s. The vibration method for determining cable tension is an inversion problem of cable dynamics. Shinke et al. (1980) [3] studied the numerical solutions for the frequency-tension curves of the eigenvalue equations of cables and provided practical formulas for evaluating the tension using the modal frequencies of the cable. However, the formulas have certain application limits and do not yield good results when the cable is not slender or is not sufficiently tensioned. Shimada and Nishimura (1988) [4] experimentally investigated the influence of the flexural stiffness of a cable on the evaluation of its tension, and they found that applying the vibration method to a short cable produces considerable errors when neglecting the flexural stiffness. Shimada (1994) [5] studied the curvilinear relationship between the high modal frequency of a cable and its tension and provided an empirical formula for evaluating the cable tension using high modal frequencies. Zui et al. (1996) [6] proposed relatively well-developed practical formulas for estimating cable tension in the form of piecewise functions, which are applicable when using the nondimensional parameter $\xi = l\sqrt{H/EI}$(H denotes the tension of a cable, EI is its flexural stiffness and l is the length of the cable) for any region of the cable. Their research complemented the work done by Shinke et al. (1980). Russell and Lardner (1998) [7] used model tests to verify the curvilinear relationship between the cable frequency and its tension in the form of nondimensional parameters.

Ren et al. (2005) [8] deduced an approximate explicit expression between the cable frequencies and tension using the energy method and hypothetical modes. They amended the approximate expression according to the numerical solution of the eigenvalue equation of the cables and provided practical formulas for estimating the cable tension. Their study provided two sets of formulas for considering the influences of the flexural stiffness and the sag effect, respectively. Fang and Wang (2012) [9] proposed formulas for estimating the cable tension according to the numerical solution of the eigenvalue equation of the cables from a different perspective. For simplicity, they neglected the sag effect by using frequencies that correspond to antisymmetric or higher modes of the cable. Huang et al. (2014) [10] proposed practical formulas for calculating the tension of vertical suspenders under hinged-fixed boundary conditions.

In addition to the sag-extensibility and flexural stiffness, the damper linked to the cable and the boundary conditions of the cable are also influential. To reduce the influence of additional dampers on the measurement precision of the cable tension, Li et al. (2009) [12] proposed an analysis method for the modal frequencies of a cable-damper system that considers cable sag. The factors that affect the cable modal frequencies were studied, including the relative height of the damper location, damping coefficient, and calculated cable length. Wang et al. (2011) [13] developed a method for estimating the tension in cables with intermediate elastic supports, and this method was applied to compute the tension in the tie bars of a concrete-filled steel tube arch bridge. Li et al. (2014) [14] proposed a real-time identification algorithm that determines the time-varying tension of stay cables using an extended Kalman filter. Wang et al. (2015) [15] deduced the equations of motion of a cable-stayed beam to consider the effects of the cable-deck interactions. The closed-form eigenvalue solution and numerical results were applied to discuss the measured frequency for estimating the cable force. Their study showed that cable-deck interactions do not affect the low-order out-of-plane frequencies of a cable-stayed beam. Yan et al. (2015) [16] presented a method for estimating the forces of cables with unknown boundary conditions. The proposed approach transforms the cable force estimation problem into a simpler problem of finding the zero-amplitude points of its mode shapes.

In recent years, an iteration method that can identify the cable tension, flexural stiffness and other system parameters was proposed. Kim and Park (2007) [17] proposed a new technique for calculating the cable tension and flexural stiffness from multiple measured natural frequencies. A finite element model (FEM) that includes both the sag-extensibility and flexural rigidity was constructed, and a frequency-based sensitivity-updating (FBSU) algorithm was applied to identify the model parameters. Park et al. (2010) [18] developed the FBSU algorithm to determine the tension of a double hanger system in a suspension bridge, which consists of two independent, parallel hangers tied with a lateral clamp. Liao et al. (2012) [19] adopted the least squares optimization scheme to eliminate errors between the calculated and measured frequencies using a precise FEM of the cable. This method also enables simultaneous identification of the cable tension and other structural parameters. Xie et al. (2014) [20] used an FEM to obtain the relation between cable frequencies and tension and utilized the genetic algorithm method to solve the inverse eigenvalue problem. This method was used to identify the tension force of hangers in arch bridges.

These studies have greatly increased the accuracy of the vibration method for measuring cable tension, especially for cables with a large sag or high stiffness. This paper develops a method for precisely determining the tension of an inclined cable with unknown boundary conditions using the sensitivity-updating algorithm. The structure of this paper is as follows: In Section 2, the nonlinear equations of motion for an inclined cable are derived. In Section 3, the finite difference method is used to develop a numerical model for cable vibration, considering the inclination angle and the rotational stiffness at two ends. In Section 4, a method is proposed for calculating the derivatives of cable frequencies with respect to the system parameters based on the derivatives of the matrix eigenvalues; this section also provides the detailed expressions of the derivatives of the mass and stiffness matrices of the cables. In Section 5, the variation in the regularity of the natural cable frequencies with the system parameters is examined, and the reliability of the method presented in Section 4 is verified. In Section 6, the FBSU algorithm for identifying the tension of an inclined cable is presented. Section 7 presents the multi-parameter identification method, which can simultaneously determine the tension, flexural stiffness, and rotational stiffness at the ends of the cable. Finally, Section 8 presents the conclusions.