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Acoustic plane wave diffraction from a truncated semi-infinite cone in axial irradiation

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ABSTRACT

The diffraction problem of the plane acoustic wave on the semi-infinite truncated soft and rigid cones in the case of axial incidence is solved. The problem is formulated as a boundary-value problem in terms of Helmholtz equation, with Dirichlet and Neumann boundary conditions, for scattered velocity potential. The incident field is taken to be the total field of semi-infinite cone, the expression of which is obtained by solving the auxiliary diffraction problem by the use of Kontorovich-Lebedev integral transformation. The diffracted field is sought via the expansion in series of the eigenfunctions for subdomains of the Helmholtz equation taking into account the edge condition. The corresponding diffraction problem is reduced to infinite system of linear algebraic equations (ISLAE) making use of mode matching technique and orthogonality properties of the Legendre functions. The method of analytical regularization is applied in order to extract the singular part in ISLAE, invert it exactly and reduce the problem to ISLAE of the second kind, which is readily amenable to calculation. The numerical solution of this system relies on the reduction method; and its accuracy depends on the truncation order. The case of degeneration of the truncated semi-infinite cone into an aperture in infinite plane is considered. Characteristic features of diffracted field in near and far fields as functions of cone's parameters are examined.

1. Introduction

The semi-infinite truncated cones are related to canonical scatterers of the acoustic field. Presently there is not enough theoretical data collected regarding its basic scattering features. The solution of diffraction problems from such scatterers provides a reliable test bed for modelling of various elements of acoustic measuring and operating systems. The aperture, obtained by cut of the cone apex, may be used to produce the necessary distribution of sound pressure. In addition, such cones may be applicable as a model of object diagnostics or for controlling the flow of fluid through an aperture.

Problems of diffraction of electromagnetic and acoustic waves on finite cones incorporating the edge have been the subject of many studies in literature [1–6]. In these publications, hollow and closed cones were examined by different asymptotic and rigorous approaches such as geometrical theory of diffraction [1], mode matching technique [2–5] and Wiener-Hopf method [6]. The finite cones in the electromagnetic case are also widely considered numerically [7,8]. For solution of the diffraction problem on finite hollow cones in [9,10] the analytical regularization procedure is developed.

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The basis of this procedure is the exact inversion of the singular part of the operator of diffraction problem. The effectiveness and the power of this approach for the solution of diffraction problem on finite acoustic cones is demonstrated in [11]. The regularization idea [9,10] is very close to the ideas which have been developed in [12–14] for electromagnetic and elastic problems.

Another class of canonical diffraction structures which has been intensively studied in acoustic and electromagnetic contexts with impedance and penetrable boundary conditions are the infinite cones [15–17] (and wedges [18,19]). Wave scattering problems for infinite truncated cones substantially have been considered in the electromagnetic case [20,21] and in the context of potential theory for the truncated cone such problem was examined in [22]. In Acoustics, a detailed analysis of the radiation features is carried on only for a limiting case when the truncated cone degenerates into an infinite plane with the circular aperture [23,24]. So in order to extend a class of problems comprising the edges in Acoustics, this paper engages with the accurate analysis of diffraction features of semi-infinite truncated cones. Some results regarding these problems have appeared in [25]. Here the regularization idea [9–11] is used to solve this kind of diffraction problem. As a result, the analytical treatment of this problem provides the following: an algorithm that is valid for any frequency range and geometrical parameters of the cone; the reliable benchmark to test out an accuracy of the purely numerical methods and the high frequency asymptotic approaches; effectiveness in computation of the diffraction characteristics rather than the computation by purely numerical method.

2. Formulation of the problem

Let us consider the acoustically soft (rigid) semi-infinite truncated cone (see Fig. 1) in the spherical coordinate system (r, θ, φ) as

$$Q_1: \left\{ r \in (c_1, \infty);\ \theta = \gamma;\ \varphi \in [0, 2\pi) \right\},$$

where c_1 is the truncation radius and γ is the cone-generating angle. The cone Q_1 is irradiated by a plane acoustic wave, which propagates along the positive direction of the z-axis and is defined as the velocity potential of unit amplitude

$$U_0(r, \theta) = \exp(ikr\cos\theta). \tag{1}$$

Here $k = \omega/c_0 = 2\pi/\lambda$ is the wave number, ω is the angular frequency, c_0 is the velocity of sound wave, λ is the wavelength. Time factor $\exp(-i\omega t)$ is suppressed throughout this paper.

We formulate the corresponding diffraction problem as a mixed boundary value problem for the Helmholtz equation

$$\nabla^2 U(r, \theta) + k^2 U(r, \theta) = 0, \tag{2}$$

with respect to the scattered potential $U(r, \theta)$, which satisfies the Dirichlet (S-case) or Neumann (R-case) boundary conditions on the conical surface in the form

$$\left[U(r, \theta) + U^{(i)}(r, \theta)\right]_{(r,\theta)\in Q_1} = 0 \quad \text{for S-case;} \tag{3a}$$

$$\partial_\theta\left[U(r, \theta) + U^{(i)}(r, \theta)\right]_{(r,\theta)\in Q_1} = 0 \quad \text{for R-case.} \tag{3b}$$

Here $U^{(i)}(r, \theta)$ is the incident field potential, which will be determined later as the field excited by the plane wave (1) in semi-infinite conical region $\{0 < r < \infty, \gamma < \theta \le \pi\}$; ∇^2 is the Laplacian defined by

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right).$$

We search for the solution of the mixed boundary value problem (2) and (3) in the class of functions that satisfy the radiation condition, as well as the finiteness of energy in any bounded volume. The second condition in our case is reduced to the fulfillment of the Meixner condition at the edge of Q_1. Due to these two additional conditions, the diffraction problem (2) and (3) is properly posed.

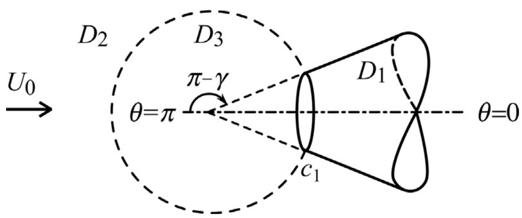

Fig. 1. Geometry of the problem.