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Influence of liquid sloshing on dynamics of flexible space structures

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ABSTRACT

This study involved an analysis of the influence of liquid sloshing on the dynamics of flexible space structures with liquid on-board by considering the main body of a spacecraft as a rigid tank, the flexible appendages as two elastic beams, and on-board liquid as an ideal liquid. The meniscus of the free surface of the liquid due to surface tension was considered. The Lagrangians of the main body of the spacecraft (rigid tank), liquid, and two beams (flexible appendages) were used in addition to assuming symmetric motion of the system; the frequency equations of the coupled system were obtained by applying the Rayleigh–Ritz method. The influence of sloshing motion on the motions of the main body and flexible appendages of the spacecraft was investigated. The results indicated that the vibration characteristics of the coupled system were dependent on the static contact angle of the liquid, irrespective of whether the angle was larger/smaller than $\theta_0 = 90°$.

1. Introduction

Large space structures vibrate easily at low frequencies because they possess low structural rigidity, given their need to be lightweight. Attitude control or orbit modification through thruster injection could cause flexible appendages such as antennae and solar arrays, as well as the liquid fuel or wastewater on the space station, to vibrate and develop strong coupled vibrations that exert a complex effect on the dynamic behaviour of the main body. This poses a serious problem for high-attitude-accuracy satellites such as those used for precise astronomical photography. Therefore, it is essential to clarify the dynamic interaction behaviour of a flexible space structure with on-board liquid in advance, to improve the stability and reliability of space structures.

Several researchers have examined the sloshing of liquids in containers in low-gravity environments theoretically. For example, in 1966, Abramson conducted a review of studies conducted up until 1966 [1]. Bauer et al. (1990 [2], 1990 [3]) conducted free vibration analyses of a liquid in a cylindrical or rectangular vessel taking into consideration the liquid meniscus due to surface tension. In 1993, Agrawal analysed the dynamic behaviour of liquid in a rotating space vehicle using a boundary-layer model [4]. In 1999, Komatsu investigated theoretically the sloshing frequency in a space vehicle tank using a mechanical model, and used potential flow models to obtain natural frequencies via a semi-empirical formula [5]. In 2002, Chiba et al. investigated the coupled natural vibration of an elastic membrane bottom and liquid in a cylindrical container with a rigid wall [6]. In 2004, Utsumi proposed mechanical models for sloshing in a tear-shaped axisymmetric tank [7]. In

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Nomenclature

A	Cross sectional area of beam
b	Length of tank
E	Young's modulus of beam
h	Equivalent liquid height: $(h_0 = h/R)$
I	Second moment of area of beam
l	Length of beam $(\lambda = l/R)$
m_f	Mass of liquid: $(\bar{m}_f = m_f/2\rho_b Al)$
m_t	Mass of rigid tank: $(\bar{m}_t = m_t/2\rho_b Al)$
M	Summation of m_f and m_t: $(\bar{M} = \bar{m}_f + \bar{m}_t)$
$\mathrm{o} - XY$	Coordinate system for spacecraft
$\mathrm{o} - r\varphi z$	Coordinate system for tank: $(\mathrm{o} - \rho\varphi\eta)$
R	Radius of rigid tank
t	Time: $(\tau = \omega_b t)$
Y_M	Displacement of rigid tank: $(y_M = Y_M/l)$
$W_i(x_i, t)$	Displacements of beams: $(w_i = W_i/l)$
$z_0\ (r)$	Static liquid free surface: $(\eta_0 = z_0/R)$
$Z(r, \varphi, t)$	Amplitude of liquid surface: $(\zeta = Z/R)$
$Z_f(r, \varphi, t)$	Displacement of liquid free surface
$\Phi(r, \varphi, z, t)$	Liquid velocity potential: $(\phi = \Phi/\omega_b R^2)$
θ_0	Static contact angle of liquid
ρ_f	Density of liquid
ρ_b	Density of beam
σ	Coefficient of free surface tension: $(\gamma = \sigma R^2 l/EI)$
$\bar{\beta}$	Area ratio of beam and tank $(=\pi R^2/2A)$
$\bar{\rho}$	Density ratio $(=\rho_f/\rho_b)$
ω_b	Natural circular frequency of beam $\left(=\sqrt{EI/\rho_b A l^4}\right)$
ξ_i	Non-dimensional coordinate: $(= x_i/l)$
ω	Coupled natural circular frequency $(\Omega = \omega/\omega_b)$

2007, Yuanjun et al. carried out a nonlinear analysis of liquid sloshing in a cylindrical container considering the static meniscus shape in low-gravity environments using an energy method under pitching excitation around the centre of gravity of the cylinder [8]. In 2007, Berglund et al. controlled the sloshing of liquid propellant in a Delta IV rocket using a pulse-suppression approach [9].

However, only a few experimental studies have focused on resolving the sloshing that occurs in low-gravity environments. In 2005, the Netherlands Agency for Aerospace, NIVR, launched a 130 kg miniature satellite called “Sloshsat Flevo” with an 87 l tank, with 33.5 l of water, to investigate the effect of sloshing behaviour on the motion of the satellite [10].

Additionally, with respect to the effects of sloshing on spacecraft motion, McIntyre et al. in 1982 revealed the relationship between the balance and stability of a flat, rotating spacecraft with liquid fuel on-board [11]. Santini et al. in 1978 and 1983 analysed the influence of motion around the centre of gravity on sloshing in orbital space structures through force balance, and discussed its stability [12,13]. Jing et al., in 2005, analysed the vibration due to liquid motion in a rectangular tank with flexible appendages subjected to pitching excitation using the energy method under conditions of microgravity and gravity [14]. Buzhinskii, in 2009, studied the effect of sloshing on rocket motion and modelled it as a thin-walled structure with liquid [15]. Recently, Farhat et al., in 2013, investigated the effect of fuel sloshing on a spacecraft and its flutter characteristics [16].

A recent study constituted the initial step in clarifying the fundamental vibration characteristics of flexible space structures with on-board liquid by proposing a mechanical model, and theoretically analysing the axisymmetric coupled vibrations of a flexible structure with on-board liquid in zero-gravity environments [17]. The proposed model involved modelling the main body as a rigid mass, flexible appendages as two elastic beams, and on-board liquid as a "spring-mass" system (mechanical model). A single liquid sloshing mode (i.e. fundamental sloshing mode) was adopted in the mechanical model, and this helped in determining the fundamental vibration characteristics of the coupled system, i.e. the main body-flexible appendages-liquid system. The present study follows on from the aforementioned study as the second step, and includes a fluid model in which the liquid is modelled as an ideal liquid considering the meniscus due to surface tension.

2. Basic equations and boundary conditions

2.1. Analytical model

In the study, the free vibrations of a spacecraft in space were considered as shown in Fig. 1. The spacecraft included flexible appendages, such as solar arrays on both sides of the main body, and the liquid on-board. The main body of the spacecraft was modelled as a rigid tank, the flexible appendages as two elastic beams, and the on-board liquid as an ideal liquid.

A rigid cylindrical tank with radius R and length b has a mass m_t and a displacement represented by Y_M in the inertia coordinate $\mathrm{o} - XY$.

The beams were modelled as uniform Euler–Bernoulli beams with length l, cross-sectional area A, density ρ_b, Young's modulus E, second moment of area I, and displacements corresponding to $W_1(x_1, t)$ and $W_2(x_2, t)$. The on-board liquid was treated as an inviscid ideal-liquid with density ρ_f and mass $m_f = \pi R^2 h \rho_f$, where h denoted liquid height when the meniscus of the liquid is ignored. The velocity potential of the liquid $\Phi(r, \varphi, z, t)$ is introduced in the coordinate system $\mathrm{o} - rz\varphi$ in which the origin is considered to be located on a flat liquid surface. In a zero-gravity condition, the surface tension was