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Investigations on bending-torsional vibrations of rotor during rotor-stator rub using Lagrange multiplier method

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ABSTRACT

The ever-increasing need of highly efficient rotating machinery causes reduction in the clearance between rotating and non-rotating parts and increase in the chances of interaction between these parts. The rotor-stator contact, known as rub, has always been recognized as one of the potential causes of rotor system malfunctions and a source of secondary failures. It is one of few causes that influence both lateral and torsional vibrations. In this paper, the rotor stator interaction phenomenon is investigated in the finite element framework using Lagrange multiplier based contact mechanics approach. The stator is modelled as a beam that can respond to axial penetration and lateral friction force during the contact with the rotor. It ensures dynamic stator contact boundary and more realistic contact conditions in contrast to most of the earlier approaches. The rotor bending-torsional mode coupling during contact is considered and the vibration response in bending and torsion are analysed. The effect of parameters such as clearance, friction coefficient and stator stiffness are studied at various operating speeds and it has been found that certain parameter values generate peculiar rub related features. Presence of sub-harmonics in the lateral vibration frequency spectra are prominently observed when the rotor operates near the integer multiple of its lateral critical speed. The spectrum cascade of torsional vibration shows the presence of bending critical speed along with the larger amplitudes of frequencies close to torsional natural frequency of the rotor. When $m \times \frac{1}{n}X$ frequency component of rotational frequency comes closer to the torsional natural frequency, stronger torsional vibration amplitude is noticed in the spectrum cascade. The combined information from the stator vibration and rotor lateral-torsional vibration spectral features is proposed for robust rub identification.

1. Introduction

Among various research issues, fault diagnosis for a rotating machine is an important topic. Once the rotor bearing system is properly designed, its continuous performance without untimely failure can be ensured by way of monitoring its health during service. Among various faults, the rotor stator rub has always been of great significance, particularly in high speed flexible rotors. Reducing the clearance for efficiency increases the undesired rotor stator contact possibilities. The rub is termed as the contact between the rotor and the stator that leads to significantly changed rotordynamic behaviour. It is one of very few causes that influence both lateral and torsional rotordynamics. The detection of torsional vibrations is also

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Nomenclature

$\mathbf{a}^{\mathrm{T}}$	unit vector along tangential direction
A^i	contact area for the *i*th nodal pair
c_g	$=g/\max(\mathrm{X}(\omega))$, clearance ratio
$\mathbf{C}_{\mathrm{r}}$, $\mathbf{C}_{\mathrm{s}}$	effective damping matrices: rotor, stator
$\mathbf{D}_{\mathrm{r}}$	damping matrix for rotor
E_s	elastic modulus of stator
$\mathbf{f}$	global force vector of body forces and surface traction
g	gap or clearance between contacting bodies
g_N, $\mathbf{g}_N$	normal gap function and the array of all the normal gap functions
$\dot{g}_T$, $\dot{\mathbf{g}}_T$	time derivative of tangential gap function and the array of all the $\dot{g}_T$
$\mathbf{G}_{\mathrm{r}}$	gyroscopic matrix for rotor
$\mathbf{G}_{\mathrm{L}}$	local contact matrix for nodal contact pair
$\mathbf{G}_{\mathrm{N}}$, $\mathbf{G}_{\mathrm{T}}$	global contact matrix for normal and tangential direction
$\mathbf{G}_{\mathrm{NT}}$	$=\mathbf{G}_{\mathrm{N}}+\mu\mathbf{G}_{\mathrm{T}}$, global normal-tangential contact matrix
$\mathbf{K}_{\mathrm{r}}$, $\mathbf{K}_{\mathrm{s}}$	stiffness matrices: rotor, stator
$\mathbf{M}_{\mathrm{r}}$, $\mathbf{M}_{\mathrm{s}}$	mass matrices: rotor, stator
$\mathbf{n}^{\mathrm{T}}$	unit vector along normal direction
N_r, N_s	degree of freedom: rotor, stator
p_N^i	normal contact pressure for *i*th contact pair
$\mathbf{P}_{\mathrm{A}}$, $\mathbf{P}_{\mathrm{B}}$	position vector for point A, B
q	number of active nodal contact pair
r	$=\omega/\omega_{L0}$, speed ratio
r_d	radius of the disc
$\mathbf{R}_{\mathrm{r}}(t)$, $\mathbf{R}_{\mathrm{s}}(t)$	external force vectors
t_T^i	scalar form of tangential surface traction for *i* th contacting pair
$\mathbf{u}_*$	displacement update without contact
$\mathbf{u}_{\mathrm{c}}$	corrective displacement vector
$\mathbf{u}_{\mathrm{r}}$, $\dot{\mathbf{u}}_{\mathrm{r}}$, $\ddot{\mathbf{u}}_{\mathrm{r}}$	displacement, velocity and acceleration vectors for rotor
$\mathbf{u}_{\mathrm{s}}$, $\dot{\mathbf{u}}_{\mathrm{s}}$, $\ddot{\mathbf{u}}_{\mathrm{s}}$	displacement, velocity and acceleration vectors for stator
$\mathbf{X}^{\circ}$	vector defining the initial gap between contacting nodal pairs
$\mathrm{X}(\omega)$	response as function of speed
$\delta\square$	variation of the parameter $\square$
λ_N, λ_T	normal and tangential contact force
$\boldsymbol{\lambda}_N$, $\boldsymbol{\lambda}_T$	normal and tangential contact force vectors
μ	friction coefficient
Ω	speed of rotation
ω_{L0}, ω_{T0}, ω_{S0}	1st natural frequency: rotor's lateral, torsional and stator's lateral direction
Π	Potential energy
Π_C^{LM}	Energy due to contact contribution

important in practical rotors where unnoticed large torsional vibrations can lead to fatigue induced cracks and failure.

The rotor stator rub has been under investigations for over three decades. Childs [1,2] in his early works has shown that the rubbing causes parametric excitation of half speed whirl at a rotor's natural frequency. He showed increase in radial stiffness and tangential force due to rubbing. In subsequent studies, third order sub-synchronous motion is also found due to partial rubbing. During rotor operation under rub, Beatty [3] emphasized the presence of second and third harmonics of the synchronous frequency. He also concluded that the frictional cross-coupling term may lead to instability during full contact rub. Choy and Padovan [4] investigated the event of rub as a nonlinear phenomenon and studied the transient motion of the rotor due to rub. The process is divided into four stages as non-contact stage, rub initiation, rub interaction and separation. They studied effects of casing stiffness, friction co-efficient, unbalance load and system damping. Muszynska [5] has provided a detailed review of the literature on the theoretical and experimental works. Muszynska and Goldman [6] in their analytical, numerical, and experimental studies, have concluded that the vibrational behaviour can be characterized by regular periodic vibrations of synchronous (1X), and sub synchronous ($\frac{1}{2}X$, $\frac{1}{3}X$...) orders. Chaotic vibration patterns were found accompanied by higher harmonics. Further, it was observed that chaotic vibration zones decrease with increase in damping. In a study of relatively new problem of wind-milling imbalance in aero engines, Groll and Ewins [7] found very rich frequency spectrum consisting of super-harmonics upto order of 9 and sub-harmonics of the order of 32.

Nonlinear vibrations related to rub have been studied by many researchers. Chu and Zhang [8] have used non-linear model with piecewise linear stiffness and have shown that rub impact between rotor and stator exhibits periodic, quasi-periodic and chaotic vibrations. In a similar study, Sun et al. [9] have shown that motion of the rotor system alternates among the periodic, chaotic and quasi-periodic vibrations, as the rotating speed increases. It was concluded that the proper increase of damping can make the chaotic motion return to periodic one, which is contrary to the observations made in [8]. Chu and Lu [10] have reported an experimental work to investigate nonlinear vibrations in a rub impact rotor system and observed a very rich form of periodic and chaotic vibrations with presence of fractional harmonic components such as $\frac{1}{2}X$, $\frac{3}{2}X$, $\frac{5}{2}X$, etc. Most of the work on the rub is based on conventional friction model. Only a few attempts are made to address the rub phenomenon in an alternate way such as impact energy model as reported by Cong et al. [11]. The stiffening effect due to rub is observed and analysed by Chu and Lu [12] and Patel and Darpe [13]. Dynamic stiffness identification for determining rub location was studied by Chu and Lu [14]. Patel and Darpe [13] have shown the possibility of detecting rub at its initiation stage. They also investigated in detail the directional nature of rub fault along with the shift in resonance speed. Advanced signal processing tools have been used for time-frequency-energy mapping to estimate the exact time of occurrence of rub and associated excitation frequencies.

Similarly, most of the rotordynamic models for rub investigations have considered lateral vibrations only. Presence of tangential force due to contact and friction leads to torsional vibrations in the system. Edwards et al. [15] have investigated