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Interaction between moving tandem wheels and an infinite rail with periodic supports – Green's matrices of the track method in stationary reference frame

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ABSTRACT

This paper approaches the issue of the interaction between moving tandem wheels and an infinite periodically supported rail and points out at the basic characteristics in the steady-state interaction behaviour and in the interaction in the presence of the rail random irregularity. The rail is modelled as an infinite Timoshenko beam resting on supports which are discretely modelling the inertia of the sleepers and ballast and also the viscoelastic features of the rail pads, the ballast and the subgrade. Green′s matrices of the track method in stationary reference frame were applied so as to conduct the time-domain analysis. This method allows to consider the nonlinearities of the wheel/rail contact and the Doppler effect. The study highlights certain aspects regarding the influence of the wheel base on the wheels/rail contact forces, particularly at the parametric resonance, due to the coincidence between the wheel/rail natural frequency and the passing frequency and also when the rail surface exhibits random irregularity. It has been shown that the wheel/rail dynamic behaviour is less intense when the wheel base equals integer multiple of the sleeper bay.

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1. Introduction

The interaction between a moving loaded wheel and a discretely supported rail is one of the railway engineering fundamental problems whose solution has applications in many practical aspects: rolling noise [1], wear of the wheel/rail rolling surfaces [2], ballast settlement and deformation in track [3,4], fault diagnosis [5], etc.

The wheel/rail interaction issue is usually the generic name for vertical interaction between a vehicle and track within the range of frequency higher than 20 Hz, which is considered the upper limit of the vehicle′s natural frequencies. When dealing with this problem, the answers to the questions related to the modelling of the vehicle-track system, including both mechanical and mathematical aspects, have to be known.

Many applications focus on the ballasted track and its response due to the interaction with a moving vehicle, where the track is deemed as an infinite periodic structure consisting of a rail resting on discrete supports including the rail pads, sleepers and ballast. This approach where only half of track is taken into account relies on the symmetry feature and, it can also be applied to the vehicle modelling. The simplest rail model is the Euler-Bernoulli beam that is extensively used when the wheel/rail interaction phenomena within the low and middle frequency range are studied or when the influence of the equidistant rail supports is neglected [6–9]. More accurate results can be however obtained by applying the Timoshenko beam model for rail, which considers the shear and the rotary inertia of the cross-section [10–13]. Consequently, the Timoshenko beam is more elastic than the Euler-Bernoulli beam and this feature mainly influences the prediction of the steady-state interaction and the pinned-pinned resonance frequency.

Most models for the rail support consist of two rigid bodies for the sleeper and ballast inertia, connected one to the other, to rail and to a rigid base, respectively, via linear elastic and damping elements that are modelling the visco-elastic features of the rail pad, the ballast and the subgrade [14,15].

As for the vehicle model, two complementary approaches are possible: one of them considers the vehicle reduced to a force traveling along the track, while the other one takes into account the degrees of freedom of the vehicle. The former approach is adequate to describe the dynamic features of the track itself or for applications which interest the propagation of elastic waves through the soil in the neighbourhood of the track [16–18]. The latter approach allows finding sufficiently accurate results in many research studies, even when the simplest models are used, such as the lumped mass model [19,20] or the two-mass oscillator [21,22]. However, more interesting results can be derived from the use of the discrete-continuous models [23,24] or the FE models [10,25].

Another issue of the wheel/rail interaction modelling refers to the excitation model which explains how the irregularity of the rolling surfaces puts the wheel/rail system into vibration. The simplest excitation model is the so-called 'moving irregularity' model which can be applied in both frequency and time-domain analysis, while considering that the wheel model is fixed above the track model and a hypothetical strip with wheel/rail irregularities is pulled at a steady velocity between the two models. This excitation model is easier to apply but unsuitable to capture the parametric excitation, due to the sleepers and the Doppler effect.

The equivalent parametric model of the track can be nevertheless used [26,27] to capture the parametric excitation derived from the sleepers. This model comes from the discrete-continuous model of the track and its mathematical form is a set of ordinary differential equations. In this case, the interaction wheel/rail problem is solved in the time-domain using the Runge–Kutta algorithm to integrate the equations of motion of the wheel and the equivalent parametric model. The Doppler effect cannot be correctly simulated when considering that the equivalent parametric model of the track is obtained in compliance with the quasi-static manner.

The 'moving mass' excitation model is more realistic since it accounts for the fact that the wheel is running along the rail with irregularities between the wheel and the rail. The time-domain analysis relies on this model, but difficulties emerge when applied in conjunction with a truncated track model based either on the modal analysis or on FEM. Indeed, such truncated models insert the boundary effects, due to the waves reflected by the model edges. This is the reason for which the models have to contain lots of sleepers to preserve the periodic feature of the ballasted track at the model centre during a sufficiently long distance, which requires a large number of differential and time consuming equations. The analysis of the wheel/rail interaction may be carried out either in the frequency-domain or in time-domain or in both. The frequency-domain analysis is easier to be done, but it requires linear models, despite that the track and wheel/rail contact have nonlinear characteristics. Such analysis is recommended in the rolling noise issues [28] and, hence, it is sometimes useful to investigate the short pitch corrugation of the rail [29].

There are many paths to follow when pursuing the time-domain analysis of the wheel/rail interaction, among which may be mentioned: the modal analysis [30], the modal sub-structuring approach [31], Fourier series approach [32] time-domain Green′s function method [15,22,24,33–36] and others.

The time-domain Green′s function method is based on the time-domain Green′s functions of the rail and wheel, which represent the response to a unit impulse of the rail and the wheel, respectively. The wheel and rail displacement in the contact point is calculated based on the contact force by applying the convolution theorem twice (for rail and wheel). It is interesting to show that the convolution theorem has been applied earlier to calculate the rail deflection due to a moving load [37,38]. The contact force is obtained when these displacements are inserted in the nonlinear algebraic equation of the wheel/rail contact and the relation is solved by iteration. Another application of the convolution integrals will allow the wheel and rail displacement in the contact point to be calculated. This method is advantageous since it can be applied along