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Dynamic behaviour of a planar micro-beam loaded by a fluid-gap: Analytical and numerical approach in a high frequency range, benchmark solutions

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ABSTRACT

Miniaturized vibrating MEMS devices, active (receivers or emitters) or passive devices, and their use for either new applications (hearing, meta-materials, consumer devices, …) or metrological purposes under non-standard conditions, are involved today in several acoustic domains. More in-depth characterisation than the classical ones available until now are needed. In this context, the paper presents analytical and numerical approaches for describing the behaviour of three kinds of planar micro-beams of rectangular shape (suspended rigid or clamped elastic planar beam) loaded by a backing cavity or a fluid-gap, surrounded by very thin slits, and excited by an incident acoustic field. The analytical approach accounts for the coupling between the vibrating structure and the acoustic field in the backing cavity, the thermal and viscous diffusion processes in the boundary layers in the slits and the cavity, the modal behaviour for the vibrating structure, and the non-uniformity of the acoustic field in the backing cavity which is modelled in using an integral formulation with a suitable Green's function. Benchmark solutions are proposed in terms of beam motion (from which the sensitivity, input impedance, and pressure transfer function can be calculated). A numerical implementation (FEM) is handled against which the analytical results are tested.

1. Introduction

In the past decades, much efforts has been put into miniaturizing acoustic elements (tubes, slits, cavities, membranes, …) used in acoustic devices (absorbers, filters, transducers, …) in order to reduce the sizes of these devices, to improve their properties or create new ones, or to lower their manufacturing cost (MEMS devices are more particularly concerned). More specifically, there has been increasing interest in designing miniaturized vibrating devices (using MEMS processes) [1], active (receivers or emitters) or passive devices, and in using them for both new applications which involve several fields of acoustics (hearing, meta-materials, consumer devices, …) and metrological purposes under non-standard conditions, namely high frequency ranges (typically up to 500 kHz), gas mixtures, and various static pressures and temperatures [2]. The miniaturisation and these new applications and purposes require deeper characterisations (analytical, numerical, and experimental) as the classical ones available until now. In this context, much of the analytic work concerns devices with

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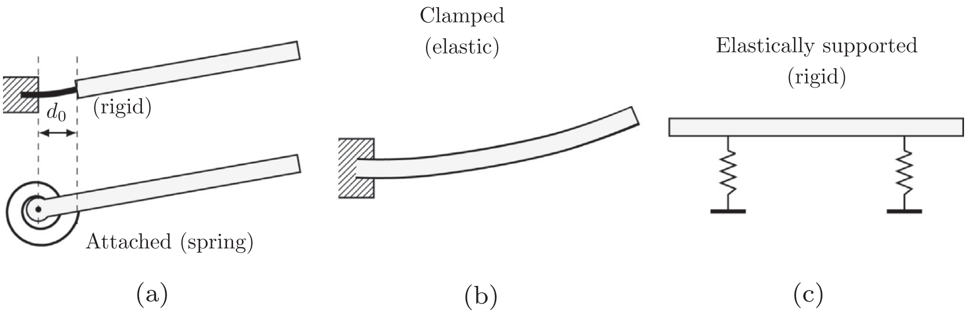

Fig. 1. Sketch of the mechanical system consisting of a rigid planar beam attached to one end of a flat spring (a), a thin elastic one-dimensional planar beam clamped to one end (b), a suspended rigid planar beam (c).

two-dimensional (circular or square) micro-structures (membranes or thin plates) for which researches into the effective design have been carried out [3,4].

The present article is concerned with the analytical and numerical approaches of the behaviour of a one-dimensional device, a planar micro-beam of rectangular shape whose length is much greater than the width and whose thickness is much lower than the width, loaded by a backing cavity or a fluid-gap (squeeze film), surrounded by very thin slits, and excited by an incident acoustic field (assumed to be uniform on the plate). The micro-beam is either a suspended rigid one-dimensional planar beam or a thin elastic one-dimensional plate clamped to one end, the other (free) end being eventually loaded by a punctual mass. The suspended rigid planar beam, which oscillates perpendicularly to its plane, is either fixed at both ends on non-rigid walls (spring-like boundaries) or attached to one end of a flat spring of negligible mass (the other end of the spring being free) (Fig. 1). The slits surrounding the beam permit both to ensure the static pressure equilibrium on both faces of the beam (role played by the vent-holes in the classical devices) and to fit to some extent the damping of the beam, along with avoiding the design of a complex suspension (note that additional damping due to non-linear phenomena which take place around the sharp edges is neglected owing to the small amplitude of the oscillations). It is worth noting that a direct coupling between the incident acoustic field and the acoustic field in the backing cavity could take place significantly, through the slits.

The literature abounds with many papers in which topics involving flexible microstructures (oscillating masses or beams, vibrating plates, membranes, or beams) and squeeze-films are of principal focus. The papers deal extensively with the viscous damping in squeeze-films which are the most significant mechanism of energy dissipation, because it is essential to understand this damping mechanism to optimise the design of these MEMS devices. The most important parameter is the Knudsen number K_n, defined as the ratio of the molecular mean free path λ_{mfp} (at the static pressure considered) to the characteristic length of the acoustic flow, namely here the thickness h_g of squeeze-film ($K_n = \lambda_{mfp}/h_g$), where the mean free path λ_{mfp} at the static pressure considered P_a is linked to the mean free path $\lambda_{mfp}^{(0)} \simeq 65$ nm at the atmospheric pressure P_0 by $\lambda_{mfp} = (P_0/P_a)\lambda_{mfp}^{(0)}$. Based on the value of this number, the flow inside the squeeze-film can be considered or not as continuous fluid flow. Many MEMS devices operate at very low pressure with a very small squeeze-film thickness ($K_n > 10$) so that the continuous fluid flow regime assumption cannot be assumed (free molecular flow regime treated in using Boltzmann equation) [5–7]. Note that the frequency involved in these papers is not much lower than the average molecular collision frequency, ranging typically from 1 MHz to 100 GHz, while in the present paper the frequency is lower than 1 MHz. The MEMS devices considered in the present paper operate at atmospheric pressure with a fluid gap thickness larger than ten micrometers ($K_n < 0.005$); therefore the fluid can be treated as continuum (and it is assumed to be Newtonian), because the mean free path is much lower than the thickness of fluid-gap, with non-slip conditions at the walls (plate or beam). Note that in the intermediate regimes, respectively slip flow regime ($0.005 < K_n < 0.1$) and transitional flow regime ($0.1 < K_n < 10$), the validity of the modelling assuming continuous flow can be extended in using the so-called "effective viscosity coefficient" which depends on the Knudsen number [8].

In the review provided in a paper published in 2003 [9], and references 6 to 23 therein, the authors point out that in much of the literature involving the situations where continuous fluid flow can be assumed, the majority of the models treat the microplate as a rigid structure, and that only few model account for flexibility of the microstructure. Moreover, in this last situation, the coupling between the fluid-film and the elastic plate is not fully modelled and in the two-dimensional situations the lateral pressure variation is neglected when the flexible microstructure structure is considered as a one-dimensional one. These analytical approaches, while appropriate to addressing specific situations, have shortcomings which prevent in several situations from obtaining accurate results, tend to preclude extension to more sophisticated problems, and inhibit more advance insight. Therefore, in the same paper, the authors present an analytical approach based on modal expansions for both the fluid-film and a flexible two-dimensional microstructure (elastic plate), which account for the strong coupling between both, assuming several boundary conditions for the plate (free or clamped edges) and for the fluid-film (pressure or displacement vanishing). Then the authors use a perturbation method to treat the problem of matching