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# Nonlinear aeroacoustic characterization of Helmholtz resonators with a local-linear neuro-fuzzy network model

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ABSTRACT

The nonlinear acoustic behavior of Helmholtz resonators is characterized by a data-based reduced-order model, which is obtained by a combination of high-resolution CFD simulation and system identification. It is shown that even in the nonlinear regime, a linear model is capable of describing the reflection behavior at a particular amplitude with quantitative accuracy. This observation motivates to choose a *local-linear* model structure for this study, which consists of a network of parallel linear submodels. A so-called fuzzy-neuron layer distributes the input signal over the linear submodels, depending on the root mean square of the particle velocity at the resonator surface. The resulting model structure is referred to as an *local-linear neuro-fuzzy network*. System identification techniques are used to estimate the free parameters of this model from training data. The training data are generated by CFD simulations of the resonator, with persistent acoustic excitation over a wide range of frequencies and sound pressure levels. The estimated nonlinear, reduced-order models show good agreement with CFD and experimental data over a wide range of amplitudes for several test cases.

## 1. Introduction

Helmholtz resonators, as schematically sketched in Fig. 1, are used in various industrial applications to absorb sound. Arrays of such resonators are applied as liners in jet engines to reduce the emission of sound to the environment [1, p. 214ff]. Combustion systems as for instance gas turbines can become thermoacoustically unstable due to the feedback between the unsteady heat release and the acoustics within the chamber. Helmholtz resonators are often inserted to stabilize the combustion process [2].

The present paper introduces a data-based reduced-order model (ROM), a so-called local-linear neuro-fuzzy network. This model is defined in the time domain, is capable of considering a change in amplitude, and can capture nonlinear effects. The data for the identification of the model parameters is generated by broadband CFD simulation. Once such a ROM is identified, it can be evaluated efficiently and may serve, e.g., as a nonlinear boundary condition in simulations of compressible flow that require acoustic boundary conditions (BC), such as computational aeroacoustics (CAA). Moreover, the methodology proposed characterizes the nonlinear resonator behavior in an efficient and robust manner, and can thus support the proper tuning of a resonator.

As mentioned above, the resonator can respond in a linear and nonlinear fashion. Linear behavior of a system means that the relation between the system input and its output can be described by a linear transfer function. A linear transfer

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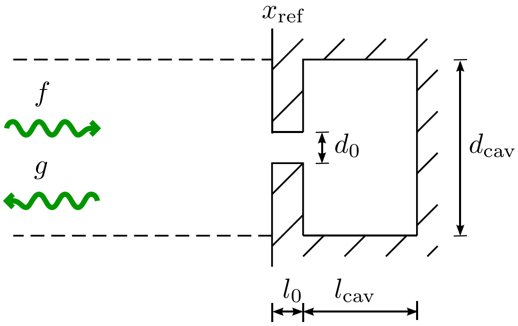

**Fig. 1.** Sketch with dimensioning of a Helmholtz resonator with acoustic waves *f* and *g*.

function can easily be transformed from the frequency/Laplace space to the time domain and vice versa by using the direct and inverse Fourier/Laplace transformation, respectively. If the resonator operates in the linear regime, its response is fully characterized by its impedance or equivalently by its reflection coefficient. Such a linear relation is valid for low acoustic amplitudes. However, when the particle velocity in the resonator neck increases beyond a certain level, the flow separates at the edges of the resonator leading to nonlinear effects as already early described by Sivian [3] as well as by Ingård and Labate [4]. The nonlinear behavior of an acoustic resonator manifests itself in several aspects: The most prominent nonlinear effect is that the harmonic behavior changes with the excitation amplitude. This effect can be modeled with an *impedance describing function*, which gives the impedance in dependency on the applied sound pressure level (SPL), see, e.g., Hersh et al. [5]. For many technical applications, the impedance describing function captures much of the dynamics for constant acoustic amplitudes. Scattering to higher harmonics [3,6,7] cannot be described by the impedance describing function, but this scattering remains on a very moderate level for Helmholtz resonators, see Förner et al. [8]. Moreover, the acoustic behavior at a certain frequency can noticeably be influenced by multi-tonal excitation, which means that in the excitation signal not only the that single frequency is present but also multiples of this frequencies, see Bodén Boden13, Boden16 and Serrano et al. [9].

Most available models are based on the first principle 1-D analysis of the fluctuating mass in the resonator neck, see, e.g., Rice [10], Cummings [7], Maa [11] as well as Hersh et al. [5]. An impedance describing function can be derived in a semi-analytical manner. Good agreement with experiments is achieved once the values of a set of correction parameters are correctly assigned. Such correction parameters are usually provided via correlation analysis and are valid for fixed amplitudes. That is why the transformation of this kind of model into time domain, where amplitudes vary dynamically, is not a trivial task.

The present methodology requires CFD simulations and broadband data analysis. Bodén [12,13] performed experimentally broadband forcing of an orifice and applied correlation analysis to separate the linear and the nonlinear contribution. Tam et al. [14] compared their direct numerical simulations (DNS) excited by a broadband signal against experiments. Tam et al. already showed in preceding studies [15,16] the potential of simulation to capture the resonator dynamics. The impact of purging flow is investigated via large eddy simulation by Mendez and Eldredge [17]. In a series of papers, Zhang and Bodony studied the effect of high amplitude excitation as well as the influence of laminar and turbulent grazing flow using the DNS approach, see e.g. [18,19].

The methodology proposed in this study applies the computational fluid dynamics/system identification (CFD/SI) approach to estimate a ROM, see Polifke [20]. Since usually a model structure is specified without considering explicitly the physics involved, such a model is called a “black-box model” in contrast to a “white-box model” derived from first principles. The model parameters are deduced from CFD time series, the so-called training data. In principle, the training data set can also be provided by experiment. This time series must cover the entire frequency range of interest for linear SI and also the entire amplitude range for nonlinear SI. Once the model structure is fixed, the model parameters are determined such that the difference between the model output and the training data is minimized in terms of a suitable norm. In order to ascertain good model performance, the estimated model is subsequently validated against an independent test data set. This methodology is applied in this study to several test cases without mean flow under ambient conditions. Since no restricting assumptions are made on the model structure, the proposed model should also be capable of characterizing more complex resonator configurations, as, e.g., in presence of purging or grazing flows, as well as other aeroacoustic devices, as for instance orifices.

Förner and Polifke [21] showed that the CFD/SI approach can provide quantitatively accurate black-box models for the Helmholtz resonator dynamics in the linear regime, i.e. for low excitation amplitudes. Only a very limited number of linear black-box models are available; the so-called output-error (OE) model has shown good performance. However, there exists a large variety of nonlinear models which can be used for SI. Representatives of nonlinear black-box models are Volterra series as well as artificial neural networks. These models are in principle capable of modeling any nonlinear behavior. However, for the test cases considered, a huge number of model parameters was required such that identification results that were achieved with reasonably long time series were not robust.

Therefore, this study introduces a “gray-box model” structure that exploits a priori knowledge of the system dynamics. Neuro-fuzzy networks offer the opportunity to incorporate such knowledge in the model structure, see, e.g., Nelles [22] or