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# Vibration measurement with nonlinear converter in the presence of noise

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ABSTRACT

Conventional vibration measurement methods use the linear properties of physical converters. These methods are strongly influenced by nonlinear distortions, because ideal linear converters are not available. Practically, any converter can be considered as a linear one, when an output signal is very small. However, the influence of noise increases significantly and signal-to-noise ratio decreases at lower signals. When the output signal is increasing, the nonlinear distortions are also augmenting. If the wide spectrum vibration is measured, conventional methods face a harmonic distortion as well as intermodulation effects. Purpose of this research is to develop a measurement method of wide spectrum vibration by using a converter described by a nonlinear function of type $f(x)$, where $x = x(t)$ denotes the dependence of coordinate $x$ on time $t$ due to the vibration. Parameter $x(t)$ describing the vibration is expressed as Fourier series. The spectral components of the converter output $f(x(t))$ are determined by using Fourier transform. The obtained system of nonlinear equations is solved using the least squares technique that permits to find $x(t)$ in the presence of noise. This method allows one to carry out the absolute or relative vibration measurements. High resistance to noise is typical for the absolute vibration measurement, but it is necessary to know the Taylor expansion coefficients of the function $f(x)$. If the Taylor expansion is not known, the relative measurement of vibration parameters is also possible, but with lower resistance to noise. This method allows one to eliminate the influence of nonlinear distortions to the measurement results, and consequently to eliminate harmonic distortion and intermodulation effects. The use of nonlinear properties of the converter for measurement gives some advantages related to an increased frequency range of the output signal (consequently increasing the number of equations) that allows one to decrease the noise influence on the measurement results. The greater is the nonlinearity the lower is noise. This method enables the use of the converters that are normally not suitable due to the high nonlinearity.



## 1. Introduction

Vibration characteristics are very important for many products in the manufacturing industry. Usually, the conventional methods of vibration measurement [1–4] use the linear properties of a physical converter. Many converters can be described by the converting function $f(x)$ independently on the physical nature of the converter. For example, in the case of capacitive converter [1,5,6] its capacitance can be expressed as:

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$$C = \frac{\varepsilon\varepsilon_0 S}{x},$$

where $\varepsilon$, $\varepsilon_0$, $S$, and $x$ are the relative static permittivity, electric constant, area of the electrode, and the distance between a vibrating object and the converter, respectively. The output signal of the eddy current sensor [5] can be expressed by a static function *f(x)*, where *x* is distance between the sensor and object, which motion is examined. In the case of photoelectric converter [7,8], which is composed of a light diode and photodiode, the output signal (current of the photodiode) is approximately proportional to $(x - \alpha_1)^{-\alpha_2}$, where $\alpha_1$, $\alpha_2$are some constants, and $x$ is the distance between the object and converter. In the case of a laser vibrometer (interferometer) [9] output signal of detector is:

$$I_{out} = I_1 + I_2 + 2\sqrt{I_1 I_2 \cos\left(\frac{2\pi}{\lambda}x\right)},$$

where $I_1$, $I_2$ - the intensities of two coherent light beams (internal reference beam and test beam), $\lambda$ - light wavelength, $x$ - path length difference between both beams, that varies with object vibration. In the range, when $x$ is changing between the neighbouring interference maxima and minima, the output signal of the detector can be considered as a monotonic nonlinear function *f(x)*.

The converter characteristic $f(x)$ may be expanded in the Taylor series at the point $x_{0f}$within the converter operation range:

$$f(x) = b_0 + b_1(x - x_{0f}) + b_2(x - x_{0f})^2 + \ldots = \sum_{k=0}^{\infty} b_k(x - x_{0f})^k, \tag{1}$$

where $b_k = f^{(k)}(x_{0f})/k!$, and $f^{(k)}(x_{0f})$ being the $k$ order derivative of $f(x)$ with respect to its argument at the point $x_{0f}$. Conventional methods use only the linear term $b_1(x - x_{0f})$.

The converters with higher linearity are considered to be more suitable for the measurement purposes. However, the ideal linear converters are not available. Practically, each nonlinear converter can be considered as a linear one, when an output signal level is very small. In this case, the influence of noise is increased significantly and signal-to-noise ratio is decreased. When the output level is increased, the nonlinear distortions are increasing simultaneously.

When the nonlinear converters are used for the measurements of wide spectrum vibration, the side effects, such as harmonic distortion and intermodulation [10], take place in conventional methods.

Nonlinearities may be treated by artificial linearization [11,12] by using an additional converter with inverse conversion function in order to compensate the nonlinear distortions. Usually, this method extends the measurement range to a very limited degree.

As a method to eliminate the nonlinear distortions, could be mentioned the method of the vibration amplitude measurement [8] in the case of harmonic vibration. According to this method, the vibration amplitude $2x_1$ is determined by the equation:

$$2x_1 = \frac{2}{b_1}(z_1 - 3z_3 + 5z_5 - \ldots), \tag{2}$$

where $z_n$ - the spectral components of the output signal $f(x(t))$ ($t$denotes time), which are found from Fourier transform of the signal. As can be seen, only one Taylor coefficient $b_1$ of the nonlinear function is required to find the amplitude.

The coefficients $b_k \quad (k = 1, 2, \ldots)$ of the nonlinear converter function $f(x)$ can be measured using method [13].

There are methods permitting to decrease the noise influence by increasing the frequency range, when signals are transmitted by radio waves. For example, the frequency modulation (wide frequency band method) in comparison with the amplitude modulation (narrow frequency band method) is characterized by improved signal-to-noise ratio [14].

Spectral methods [15] are used to solve numerically the nonlinear differential equations, involving the Fourier transform. The solution is written as a sum of certain "basic functions" (for example, as a Fourier series), then the coefficients in the sum are calculated till the differential equation becomes satisfied. Spectral techniques [16] are used for the identification of vibration parameters of absorber, which is described by a nonlinear differential equation. These parameters are determined by the Gauss-Newton least-squares algorithm [17]. In the literature [18], it is described the least-squares technique used for computing the free transverse vibration of membranes and plates. However, it is presumed that used vibration converters are linear.

Vibration parameters of the physical model with complicated nonlinear relationships can be identified using neural network that should be trained, although its efficiency is not guaranteed [19,20].

In this paper, the spectral technique is applied to derive the system of equations for the proposed method of vibration measurement, when converter is described by a monotonic nonlinear function *f(x)*. The parameter *x(t)* describing vibration is expressed as Fourier series. Then, the spectral components of the converter output *f(x(t))* are determined using Fourier transform. Obtained system of nonlinear equations is solved by using the least-squares technique (Newton-Gauss and Levenberg-Marguard algorithms) that permits to find the solution in the presence of noise.

This paper aims to fill the gap in the measurements of wide spectrum vibration using nonlinear converters with monotonic nonlinear characteristic *f(x)* in the presence of noise. The novelty of the proposed method consists of increased