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On strain-rate independent damping in continuum mechanics

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ABSTRACT

Strain-rate independent damping is a theory of energy dissipation in solids. It is based on the assumption that an increase or decrease in the strain-energy density correlates with a multiplication of $1+\eta$ or $1\text{-}\eta$ respectively, of the material stiffness matrix, with $0 \leq \eta \ll 1$ with η either a constant or a function of the strain-energy density.

This type of damping has a loss (Watt m^{-3}) of η times the absolute value of the rate of change of the strain-energy density. For uni-axial strain and a suitable function of the strain-energy density, the energy dissipation (Joule m^{-3}) due to an infinitesimal change of the strain is strain-rate independent and proportional to the absolute value of the strain raised to a power ranging from 1 to 2. This is an idealization of tests results, based on forced harmonic strain cycles, with an energy dissipation (Joule m^{-3} cycle^{-1}) found to be nearly frequency independent and almost proportional to the strain amplitude raised to a power ranging from 2 to 3.

The PDEs derived for strain-rate independent damping can be solved for 1, 2 or 3 dimensions via direct integration, provided that the software supports PDE coefficients that are functions of the solution and its space and time derivatives. A 3D problem with 22,000 DOF's and 10,000 time steps was solved successfully and convincingly.

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1. Introduction

This paper focuses on energy dissipation in a group of commonly used construction materials such as steel, iron, aluminum, bronze, glass, wood, soil and many more. It is typical for these materials that the energy dissipation (Joule m^{-3}) is independent of the rate of change of the deformation or at least nearly so. An important concept used throughout is *damping* and since this concept is not unambiguous the following definitions are given together with the SI units.

Strain-rate independent damping (SI-damping) is a material property; it has an energy dissipation (Joule m^{-3}) due to an infinitesimal change of the deformation which is independent of the rate of change of the deformation.

Frequency-independent damping is the energy dissipation (Joule cycle^{-1}) of an object made of a material with strain-rate independent damping, vibrating in a single-mode.

Linear-viscous damping is a material property; it has an energy dissipation (Joule m^{-3}) due to an infinitesimal change of the deformation which is proportional to the rate of change of the deformation.

Hysteretic damping is the energy dissipation (Joule cycle^{-1}) of an object made of a material with linear-viscous damping, vibrating in a single mode and divided by a number proportional to the frequency of that mode.

SI-damping and linear-viscous damping are assumed to be first principles because: "The physics of damping is very diverse and not well understood. In spite of a large amount of research, understanding of damping mechanisms is quite primitive" [1].

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The purpose of this paper is to investigate the behavior of vibrating continuous objects made of a material with SI-damping and to compare it with other damping models.

It has been observed that in a significant category of commonly used construction materials the energy dissipation (Joule m^{-3}) per strain-cycle in the case of uni-axial strain is only a weak function of the frequency, and nearly proportional to the strain amplitude ε_1 raised to a power n with n ranging from 2 to 3 [1,2]. This article is based on the idealization that the energy dissipation is exactly frequency independent and proportional to ε_1^n. Frequency-independent damping, however, is defined for single modes only because two or more modes will lead to mode erosion (transition to a non modal shape), to be shown in Section 3. This fact excludes the possibility of mode superposition, leaving direct integration open as the only option to solve the PDEs.

Frequency-independent damping is therefore replaced by its underlying first principle, SI-damping, as follows: the energy dissipation (Joule m^{-3}) due to an infinitesimal change of the strain $\partial\varepsilon$ is independent of $\partial\varepsilon/\partial t$ and proportional to $|\varepsilon|^{n-1}$. The exponent $n-1$ rather than n is explained by integrating $(|\varepsilon|^{n-1})\mathrm{d}\varepsilon$ from $-\varepsilon_1$ to $+\varepsilon_1$ resulting in $(2/n)\varepsilon_1^n$, which is proportional to the idealized energy dissipation per strain cycle above. SI-damping for multi-axial strains is a generalization of SI-damping for uni-axial strains but formulated in terms of strain-energy density and its rate of change. The 3D formulation is provided in Section 8.

Many materials feature approximate SI-damping but certainly not all. Exceptions include polymers [2], certain alloys with high damping at particular frequencies [3] or materials with a non-negligible strain-stress history. These materials will not be addressed in this article.

When $n=2$ and the amplitude of an object vibrating in a single mode halves after e.g. 100 cycles, the amplitude of any other object, made of the same material, will also halve after 100 cycles regardless of its frequency or initial amplitude. Another 100 cycles will halve the amplitude again. However, all other conditions being equal, when $n>2$, the amplitude decay may be faster depending on amplitude but will always be slower eventually. Other damping models fail to show these properties:

An object made of material with *linear-viscous damping* (see Fig. 1a below) does not reflect it as its energy dissipation per cycle is proportional to the frequency instead of being frequency independent.

Hysteretic damping attempts to provide a solution by dividing the energy dissipation for each mode by a number proportional to the frequency of that mode. This might seem to make the energy dissipation per cycle frequency-independent but linear models with imposed frequency-independent energy dissipation are inconsistent as will be discussed below in Section 7 and summarized in the last conclusion of Section 9.

Energy-dissipation per cycle in an object made of material with *Coulomb damping* (Fig. 1b) is frequency-independent but is proportional to the amplitude, so $n=1$, well outside the range 2 to 3, resulting in linearly declining amplitude in the case of free vibrations. Coulomb damping will result in rigid body behavior if the stress amplitudes are small.

To overcome these problems the present article will argue in favor of a friction-based constitutive model (Figs. 1c and 1d).

Note that the strain u/L, where L is the relaxed length of the spring, is associated with the SDOF objects of Fig. 1 and the strain ε or $\partial u/\partial x$ with 1D continuous objects. The strain $\mathbf{Du}$ is associated with 2D and 3D continuous objects where $\mathbf{D}$ is the differential operator matrix and $\mathbf{u}$ the displacement vector. Expanded versions of $\mathbf{D}$ and $\mathbf{u}$ to be provided in Section 8.

The mechanisms represented in Figs. 1c and 1d predict frequency-independent damping, because the friction force of a Coulomb damper is independent of the speed $\mathrm{d}u/\mathrm{d}t$ over the damper. The energy dissipation per cycle then follows from integrating $(|u/L|^{n-1})\mathrm{d}u$ from $-u_1$ to $+u_1$ and is therefore proportional to u_1^n where u_1 is the amplitude. The very same mechanisms also predict SI-damping because the energy-dissipation due to a change of strain $\mathrm{d}u/L$ is independent of the rate of strain of the spring $(\mathrm{d}u/\mathrm{d}t)/L$. SI-damping is not a new principle: if during the entire cycle the energy dissipation is always strain-rate independent then the energy dissipation (Joule m^{-3} cycle^{-1}) is frequency-independent.

The calculations in this article will be based on SI-damping with $n=2$ as in Fig. 1c but SI-damping with $n\neq 2$ will be briefly considered in Section 4; however, the formulation of the equations will always reflect the general case of SI-damping with $n\geq 2$.

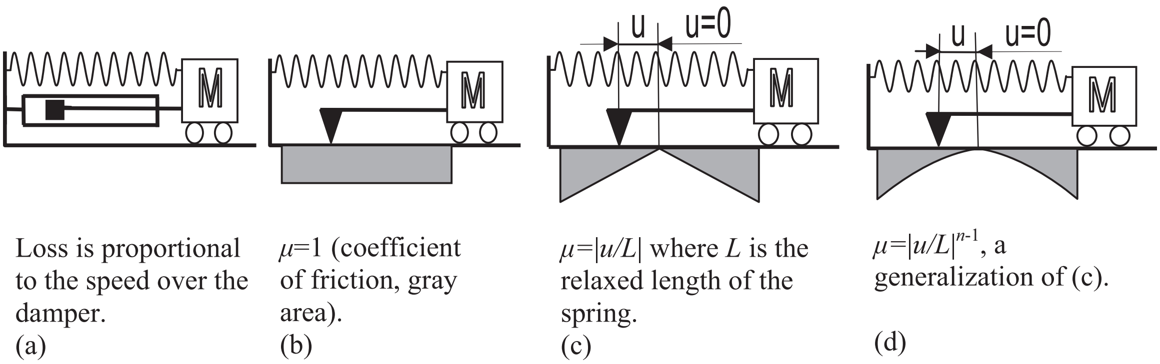

Fig. 1. Constitutive models of four types of material damping. (a) Linear-viscous damping; (b) Coulomb damping; (c) SI-damping with $n=2$; (d) SI-damping with $n\geq 2$.

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