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Direct response and force transmissibilities in the characterization of coupled structures

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ABSTRACT

A description of coupled structures, and structures with blocked degrees of freedom, is presented in terms of direct transmissibility functions (DTFs) as an alternative to the standard mobility approach. The notion of DTFs between degrees of freedom is extended to DTFs between groups of subsystems for that purpose. The relation between DTFs and mobilities is clearly established and the concept of direct force transmissibility, which offers a different description of signal transmission within a structure, is also introduced. Moreover, a physical interpretation for the decomposition of the mobility matrix of the coupled substructures in terms of Schur complements is given. This highlights the transmission process from input forces to substructure responses. Finally, two numerical examples are included to elucidate all concepts and outline their potential for practical applications. The first one deals with predicting vibrations of a plate after blocking some of its degrees of freedom. The second one addresses signal transmission between two plates with elastic coupling.

1. Introduction

Mobility and impedance methods play a central role in structural dynamics [1] and are frequently used in the vibroacoustic characterization of assembled structures, and in transmission path analysis (TPA) as well. With regard to the former, the basic problem consists in finding methods to predict the level of vibration and/or structure-borne noise that one will find when connecting a possible vibroacoustic source (e.g., auxiliary equipment) to a given structure. In practice, the source and the structure are usually built by different manufacturers so there is a need to find a methodology to predict the results of their assembly (think, for instance, of the case of a railway coach or an aircraft, where external sources manufactured by sub-suppliers, like the engines or the air conditioning system, are to be connected). Well-known methodologies that deal with this situation are those relying on the concepts of free velocity and blocked forces [2,3], or power-based formulations like the source description method [4]. These approaches have some inherent difficulties given that the measurements on sources are to be made with free boundary conditions (the sources are to be suspended without rigid connections), which is not always feasible (especially for large sources). In addition, large test rigs may also be necessary for the measurement of blocked sources. It is of no surprise then, that recent efforts have focused on the possibility of characterizing structures without needing to dissemble them, either by resorting to virtual prototype modeling strategies [5,6] or to methods which only require in situ measurements [7–10]. Though most methods deal with point connections linking the source and the structure, it is worthwhile mentioning recent work addressing the case of line connections as well [11].

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Mobilities are also central to classical TPA [12–15]. In classical TPA one aims at factorizing the response of any considered degree of freedom (d.o.f) of a structural system in terms of the contributions of the external loads acting on it. In practice, the method consists of a two-step procedure in which mobilities are measured in a first step, with the system being non-operative, and then operational loads (mechanical forces and/or volume velocities) get recorded in a second step. Once all measured data are obtained, the postprocessing to obtain the force contributions is rather straightforward. However, classical TPA is well-known to suffer from some drawbacks, such as the active parts of the system having to be removed when measuring mobilities, and operational forces being extremely difficult to obtain for some systems (see e.g., [16]). To avoid those measurement difficulties, as well as to deal with those situations in which it is not possible to act on the external loads to solve a particular vibroacoustic problem, methods based on transmissibility measurements were developed from the very first beginnings of TPA [17]. In [17], the basis of the GTDT (Global Transmissibility / Direct Transmissibility) approach to TPA (as lately termed in [18]) was established. The method factorizes the response at any d.o.f in terms of the responses at other system d.o.f.s, rather than in terms of external loads, and only involves in-situ, straightforward measurements with no demounting tests. The price to be paid is that the postprocessing is slightly more involved as it relies on blocked (originally termed *direct*) transmissibilities (DTFs) between d.o.f.s. These DTFs cannot be directly measured but they are to be computed from standard measured transmissibilities (*global* transmissibilities, GTFs) [17–19]. Although formerly developed as experimental methods, classical TPA and the GTDT method have also been applied to numerical models (see e.g., [20,21]).

Comparisons between the performance of various TPA methods are available in literature. Classical TPA is compared with the GTDT approach in [19,22] and with the in-situ method which is described in [7]. The TPA approach in [7] has also been compared with the GTDT method in [23]. It is also worth mentioning that, throughout the past few decades, considerable interest has been put on operational TPA [24–31] to reduce the time cost and measurements required by two-step approaches such as the classical TPA or GTDT methods. However, operational TPA is not free of problems. For instance, if some path is missing its contribution is automatically assigned to other paths, so special care has to be taken when applying them [16]. Hybrid approaches have also been considered [32]. Classical and operational TPA are contrasted in [33]. A detailed review exposition and comparison of existent TPA methods can be found in [34] (see also [35,36]).

Several works have been carried out exploiting the possibilities of DTFs. For instance, the concept of direct transmissibility was applied to continuous systems in [18,37], to discrete systems made of masses, dampers and stiffeners in [19] (see also the related work in [38]) and to finite element models [21]. Experimental results have been reported on a controlled simple mechanical system [22], whereas descriptions on some industrial applications can be found in [39,40]. DTFs have been also linked to energy transmission paths in statistical energy analysis [41], and graph theory has been used to compute them [42–45]. They are also at the basis of path computation in SmEdA (Statistical modal Energy distribution Analysis) models [46] and in energy distribution models [47]. The recent work in [48] opens the door to extend graph theory path analysis to low frequency models. DTFs can also be used to experimentally determine SEA (Statistical Energy Analysis) coupling loss factors without having to measure the power input into subsystems [49].

In this paper we will explore some further possibilities and results for DTFs. The paper is essentially theoretical, and potential applications of the presented developments for the experimental characterization of assembled structures will be only outlined throughout text. A typical case would be that of an active part of a system (e.g., an engine car or an auxiliary equipment of a train) being connected to a passive part (e.g., car chassis or train coach) through resilient or stiff mounts. In particular, we will be interested in establishing the following theoretical results:

- show the mathematical connection between mobilities and DTFs,
- extend the concept of response DTFs between two d.o.f.s in a system, to that of DTFs between groups of d.o.f.s in the system,
- introduce the notion of force DTFs, which provides an alternative to describe vibration transmission in a built-up structure,
- present a general framework that encompasses both, the force and response DTF approaches. This can be done by factorizing the system mobility matrix in terms of its Schur complements.

A preliminary version of this work with the mathematical basic relations was presented by the authors in [50]. A more in-depth analysis is given in this work with the inclusion of a detailed new example to envision the potential of the method, when applied to assembled structures. The example consists of two plates linked by a set of elastic springs, and we show how DTFs between groups of d.o.f.s and force DTFs can be applied to it. The paper is organized as follows. Section 2 first contains an overview of the mobility, global transmissibility and direct transmissibility approaches to TPA. It establishes the connection between them and proceeds to the definition of direct transmissibilities between sets of d.o.f.s. Direct force transmission is presented in Section 3, whereas Section 4 provides the general framework for physical interpretation in terms of Schur complements of the mobility matrix. Numerical examples are presented in Section 5 and conclusions close the paper in Section 6.