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Quasiperiodic energy harvesting in a forced and delayed Duffing harvester device

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ABSTRACT

This paper studies quasiperiodic vibration-based energy harvesting in a forced nonlinear harvester device in which time delay is inherently present. The harvester consists of a delayed Duffing-type oscillator subject to a harmonic excitation and coupled to a piezoelectric circuit. We consider the case of a monostable system and we use perturbation techniques to approximate quasiperiodic responses and the corresponding averaged power amplitudes near the primary resonance. The influence of different system parameters on the performance of the quasiperiodic vibration-based energy harvesting is examined and the optimal performance of the harvester device in term of time delay parameters is studied. It is shown that in the considered harvester system the induced large-amplitude quasiperiodic vibrations can be used to extract energy over broadband of excitation frequencies away from the resonance, thereby avoiding hysteresis and instability near the resonance.

1. Introduction

In vibration-based energy harvesting (EH) systems subject to a harmonic excitation EH performance is considerably limited when the harvester device operates in a linear regime. This is because the natural frequency of the mechanical subsystem must always match the fundamental frequency of the excitation [1–3]. To overcome such a limitation nonlinear attachments is often used to substantially extend the bandwidth of the harvester over a broadband of excitation frequencies, either in the case of monostable harvester devices with hardening characteristic [4–7] or in the case of bistable ones [6,8–11]. However, exploiting nonlinear attachments in the harvester gives rise to hysteretic behavior in the frequency response near the resonance [12] and therefore the problem of instability of the response remains. To circumvent such instabilities near the nonlinear resonance, the idea of exploiting quasiperiodic (QP) vibration away from the resonance to extract energy in broadband of frequency has been proposed [13,14].

Yet, in certain harvester systems under aerodynamic and base excitations, it was shown that QP vibrations cause a substantial reduction in the harvested power [15,16] beyond the flutter speed and then extracting energy from such systems, QP vibrations should be avoided. Nevertheless, it was demonstrated recently that in the presence of time delay QP vibrations can have a beneficial effect on the EH performance [14]. Indeed, in a delayed van der Pol-type harvester system under delay amplitude modulation, the induced large-amplitude QP vibrations occurring in broad range of parameters were

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used to extract energy with good performance. The idea of using time delay was also used to extend the dynamic range of an energy harvester with nonlinear damping [17] demonstrating that this concept can provide substantial performance in the EH capacity.

Taking advantage from using QP vibrations to extract energy from a delayed self-excited harvester system [14], the present work explores QP vibration-based EH in a delayed Duffing harvester device subject to harmonic external excitation. This study can be useful in certain applications for which a delayed state feedback is present in the mechanical subsystem of the harvester. For instance, in milling and turning operations the inherent time delay in the position commonly arises in the process [18–21] such that the time delay is not considered as an additional input power of the harvester. However, in applications where the time delay is introduced as an input power, the problem of energy balance between the generated and the consumed powers should be examined [17]. On the other hand, the forced Duffing oscillator is usually adopted in the model of orthogonal cutting [22–24,21] for which the harmonic forcing is generated by the cutting process.

Although the dynamics of a forced delayed Duffing oscillator has been largely studied in the literature [25–29], the exploitation of the induced QP vibrations to extract energy remains missing, which forms the objective of the present work. This paper will first present the harvester system in the next section. The periodic response and the output average power are then approximated using the multiple scales method. In Section 3, the QP response is obtained applying the second-step multiple scales method and the corresponding harvested power is examined. The influence of different system parameters of the harvester device on the EH performance is analyzed in Section 4 and a summary of the results is given in the concluding section.

2. Model description and periodic energy harvesting

Consider an energy harvester system consisting in an excited Duffing oscillator coupled to an electrical circuit through a piezoelectric device. We assume that the mechanical component of the harvester is under an inherent time delayed in the position such that the governing equation for the harvester can be written in the dimensionless form as

$$\ddot{x}(t) + \delta\dot{x}(t) + \omega_0^2 x(t) + \gamma x(t)^3 - \chi v(t) = \alpha x(t-\tau) + f\cos(\lambda t) \tag{1}$$

$$\dot{v}(t) + \beta v(t) + \kappa\dot{x}(t) = 0 \tag{2}$$

where $x(t)$ is the relative displacement of the rigid mass m, $v(t)$ is the voltage across the load resistance, δ is the mechanical damping ratio, γ is the stiffness parameter, χ is the piezoelectric coupling term in the mechanical attachment, κ is the piezoelectric coupling term in the electrical circuit, β is the reciprocal of the time constant of the electrical circuit, f and λ are, respectively, the amplitude and the frequency of the excitation, while α and τ are, respectively, the feedback gain and time delay. It is assumed that time delay is inherently present in the harvesting system, as in the milling and turning operations [18–21] for which Eq. (1) with $\chi = 0$ is commonly used to model such processes. The case where the time delay is included as a control within the electromagnetic coupling was considered recently in [30]. Note that the dynamics of the delayed Duffing oscillator in the absence of the piezoelectric coupling (Eq. (1) with $\chi = 0$) has been examined in details [28,29], including bifurcation and stability of periodic and QP solutions Fig. 1.

To investigate the response of the harvester system (1), (2) near the primary resonance we suppose the resonance condition $\lambda = \omega_0 + \sigma$ where σ is a detuning parameter. The method of multiple scales [31] is implemented by introducing a bookkeeping parameter ϵ and scaling parameters as $\delta = \epsilon\tilde{\delta}$, $\gamma = \epsilon\tilde{\gamma}$, $\chi = \epsilon\tilde{\chi}$, $\alpha = \epsilon\tilde{\alpha}$, $f = \epsilon\tilde{f}$, $\sigma = \epsilon^2\tilde{\sigma}$. Thus, Eqs. (1), (2) read

$$\ddot{x}(t) + \omega_0^2 x = \epsilon(-\tilde{\delta}\dot{x}(t) - \tilde{\gamma}x(t)^3 + \tilde{\chi}v(t) + \tilde{\alpha}x(t-\tau) + \tilde{f}\cos(\lambda t)) \tag{3}$$
$$\dot{v}(t) + \beta v(t) + \kappa\dot{x}(t) = 0 \tag{4}$$

Applying the multiple scales method [31] one obtains the modulation equations

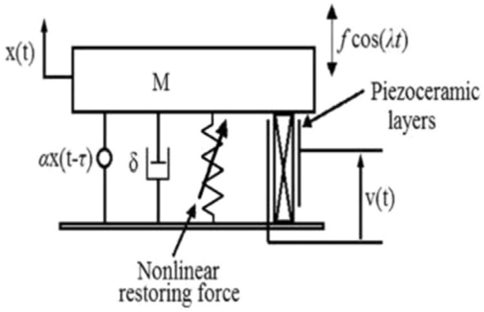

Fig. 1. Schematic description of the EH system.