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A general theory for bandgap estimation in locally resonant metastructures

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ABSTRACT

Locally resonant metamaterials are characterized by bandgaps at wavelengths that are much larger than the lattice size, enabling low-frequency vibration attenuation. Typically, bandgap analyses and predictions rely on the assumption of traveling waves in an infinite medium, and do not take advantage of modal representations typically used for the analysis of the dynamic behavior of finite structures. Recently, we developed a method for understanding the locally resonant bandgap in uniform finite metamaterial beams using modal analysis. Here we extend that framework to general locally resonant 1D and 2D metastructures (i.e. locally resonant metamaterial-based finite structures) with specified boundary conditions using a general operator formulation. Using this approach, along with the assumption of an infinite number of resonators tuned to the same frequency, the frequency range of the locally resonant bandgap is easily derived in closed form. Furthermore, the bandgap expression is shown to be the same regardless of the type of vibration problem under consideration, depending only on the added mass ratio and target frequency. For practical designs with a finite number of resonators, it is shown that the number of resonators required for the bandgap to appear increases with increased target frequency, i.e. more resonators are required for higher vibration modes. Additionally, it is observed that there is an optimal, finite number of resonators which gives a bandgap that is wider than the infinite-resonator bandgap, and that the optimal number of resonators increases with target frequency and added mass ratio. As the number of resonators becomes sufficiently large, the bandgap converges to the derived infinite-resonator bandgap. Furthermore, the derived bandgap edge frequencies are shown to agree with results from dispersion analysis using the plane wave expansion method. The model is validated experimentally for a locally resonant cantilever beam under base excitation. Numerical and experimental investigations are performed regarding the effects of mass ratio, non-uniform spacing of resonators, and parameter variations among the resonators.

1. Introduction

Inspired by photonic crystals in electromagnetism, researchers have long investigated phononic crystals for their potential to filter or redirect elastic waves [1]. Phononic crystals exhibit bandgaps (i.e. frequency ranges where elastic or

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Nomenclature

B_i	Boundary differential operator of order $[0, 2p-1]$
D	One or two-dimensional domain of the system
D_j	Subdomain of D containing the position of the jth resonator
$F(\mathbf{P}, s)$	Laplace transform of $f(\mathbf{P}, t)$
$H_r(s)$	Laplace transform of $\eta_r(t)$
$M(j\omega)$	Effective modal dynamic mass
N	Number of modes used in the discretization of the field variable w
$P_{\mathcal{L}}$	Polynomial associated with the stiffness operator $\mathcal{L}$
$Q_r(s)$	Laplace transform of $q_r(t)$
S	Total number of resonators on the structure
S_{opt}	Number of resonators that gives the greatest effective bandgap width $\Delta\omega(S)$
$U(\mathbf{P}, t)$	Laplace transform of $u(\mathbf{P}, t)$
$U_j(s)$	Laplace transform of $u_j(t)$
$W_1(\mathbf{G}_m)$	Plane wave amplitude of the primary structure associated with the mth reciprocal lattice vector $\mathbf{G}_m$
$W_b(s)$	Laplace transform of $w_b(t)$
ΔD_j	Characteristic size of the subdomain D_j
$\Delta\omega(S)$	Effective bandgap width with S resonators on the structure
$\Delta\omega_\infty$	Bandwidth of the infinite-resonator locally resonant bandgap
Ω_r	Normalized rth structural resonant frequency
$\Psi_r(s)$	Laplace transform of $\psi_r(t)$
$\delta(\mathbf{P})$	Dirac delta function
δ_{ij}	Kronecker delta
$\eta_r(t)$	Modal coordinate for the displacement $w(\mathbf{P}, t)$ corresponding to the rth mode of the plain structure
$\hat{\omega}$	Normalized excitation frequency
$\mathcal{L}$	Stiffness differential operator of order $2p$
μ	Ratio of resonator mass to plain structure mass
ω	Excitation frequency
ω_r	rth resonant frequency of the plain structure
ω_t	Identical target frequency of all of the resonators on the structure
$\omega_{res,j}$	Natural frequency of the jth resonator
∂D	Boundary of the domain D
$\phi_r(\mathbf{P})$	rth mode shape of the plain structure
$\psi_r(t)$	Modal coordinate for the resonator displacement field $u(\mathbf{P}, t)$ corresponding to the rth mode of the plain structure
σ	Standard deviation in resonator natural frequencies
$\mathbf{G}_m$	mth reciprocal lattice vector
$\mathbf{P}$	Position of a point in the domain
$\mathbf{P}_j$	Position of the jth resonator
$\mathbf{k}$	Bloch wavevector
$f(\mathbf{P}, t)$	External forcing in the domain
k_j	Stiffness of the jth resonator
$m(\mathbf{P})$	Mass density at a point in the domain
m_j	Mass of the jth resonator
$m_{p,j}$	Point mass at the attachment location of the jth resonator
n_{trials}	Number of random sets of normal random distributions of resonator natural frequencies
p	Integer defining the order of the governing partial differential equation
$q_r(t)$	Modal excitation of the rth mode
s	Complex Laplace variable
$u(\mathbf{P}, t)$	Displacement of the continuous resonator field at a point in the domain
$u_j(t)$	Relative displacement of the jth resonator
$w(\mathbf{P}, t)$	Displacement of a point in the domain
$w_b(t)$	Base motion of the structure

acoustic waves cannot propagate) produced by Bragg scattering [2–4], which occurs when the wavelength of the incident wave is on the order of the lattice constant of the crystal [5,6]. Therefore, a fundamental limitation of Bragg-based phononic crystals is that it is only possible to create low-frequency bandgaps using very large structures. In their seminal work, Liu et al. [7] showed the potential for locally resonant metamaterials to create bandgaps at wavelengths much larger than the lattice size, enabling the creation of low-frequency bandgaps in relatively small structures. Locally resonant metamaterials contain resonating elements, whether mechanical [7,8] or electromechanical [9–11], which are capable of storing and transferring energy. A significant body of research has examined locally resonant elastic/acoustic metamaterials of various types. Ho et al. [8] examined a similar system to Liu et al. [7] using a rigid frame with rubber-coated metal spheres as resonators. For that same type of system, Liu et al. [12] found analytic expressions for the effective mass densities of 3D and 2D locally resonant metamaterials, showing that the effective mass becomes negative near the resonant frequency. Simplifying the analysis, others have used lumped-mass models to obtain the locally resonant bandgap [13,14]. Other researchers have studied different implementations of resonators for different types of elastic waves [15–21], and two-degree-of-freedom resonators [22]. Moving towards analytical predictions for the bandgap edge frequencies, Xiao et al. [23] used the plane wave expansion method to study flexural waves in a plate with periodically attached resonators, giving a method to predict the edges of the bandgap. Peng and Pai [24] also studied a locally resonant metamaterial plate, finding an explicit expression for the bandgap edge frequencies.

Much of the research on locally resonant metamaterials has relied on unit-cell based dispersion analysis, using techniques such as the plane wave expansion method to obtain the band structure of the metamaterial. This type of analysis lacks the information of modal behavior and cannot readily answer questions such as the dependence of the bandgap width on the number and spatial distribution of attachments in a finite structure. To this end we recently presented [25] a modal