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Five degrees of freedom linear state-space representation of electrodynamic thrust bearings

J. Van Verdeghem*, V. Kluyskens, B. Dehez

Center for Research in Mechatronics (CEREM), Institute of Mechanics, Materials and Civil Engineering (IMMC), Université catholique de Louvain (UCL), Louvain-la-Neuve 1348, Belgium

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ABSTRACT

Electrodynamic bearings can provide stable and contactless levitation of rotors while operating at room temperatures. Depending solely on passive phenomena, specific models have to be developed to study the forces they exert and the resulting rotordynamics. In recent years, models allowing us to describe the axial dynamics of a large range of electrodynamic thrust bearings have been derived. However, these bearings being devised to be integrated into fully magnetic suspensions, the existing models still suffer from restrictions. Indeed, assuming the spin speed as varying slowly, a rigid rotor is characterised by five independent degrees of freedom whereas early models only considered the axial degree. This paper presents a model free of the previous limitations. It consists in a linear state-space representation describing the rotor's complete dynamics by considering the impact of the rotor axial, radial and angular displacements as well as the gyroscopic effects. This set of ten equations depends on twenty parameters whose identification can be easily performed through static finite element simulations or quasi-static experimental measurements. The model stresses the intrinsic decoupling between the axial dynamics and the other degrees of freedom as well as the existence of electrodynamic angular torques restoring the rotor to its nominal position. Finally, a stability analysis performed on the model highlights the presence of two conical whirling modes related to the angular dynamics, namely the nutation and precession motions. The former, whose intrinsic stability depends on the ratio between polar and transverse moments of inertia, can be easily stabilised through external damping whereas the latter, which is stable up to an instability threshold linked to the angular electrodynamic cross-coupling stiffness, is less impacted by that damping.

1. Introduction

Magnetic bearings ensure contactless guiding of rotors through electromagnetic forces. In the absence of contact, there is no friction and thus no mechanical wear, allowing to increase the bearings' lifetime as well as greatly diminishing losses. They are thus attractive and even more compelling in very high speed and vacuum applications, compared to conventional solutions such as ball or hydrostatic bearings. Magnetic bearings can be either active (AMB) or passive (PMB) [1]. Nowadays, AMBs represent the vast majority of magnetic bearings implemented in industrial applications and employ electromagnets

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* Corresponding author.
E-mail address: joachim.vanverdeghem@uclouvain.be (J.V. Verdeghem).

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Nomenclature

Latin characters

$\mathbf{A}$	State matrix
$\mathbf{B}$	Input matrix
C	External damping factor
F	Force
$\mathbf{I}$	Currents in the windings
J_p	Polar moment of inertia
J_t	Transverse moment of inertia
k	Stiffness
K	Flux linkage proportionality coefficient
l	Distance between the EDTB geometrical center and the rotor center of mass
$\mathbf{L}$	Self and mutual inductance coefficients
L_c	Cyclic inductance
m	Rotor mass
N	Number of phases
p	Number of pole pairs of the magnetic field
$\mathbf{P}$	Transformation matrix
q	EDTB geometrical center transverse displacements in complex coordinates
$\mathbf{Q}$	Transformation matrix
R	Winding resistance
T	Torque
$\mathbf{T}$	Torque in matrix form
$\mathbf{U}$	Transformation matrix
$\mathbf{v}$	State vector
x	EDTB geometrical center X-axis displacement
y	EDTB geometrical center Y-axis displacement
z	EDTB geometrical center axial displacement

Superscripts

$'$	Related to rotor center of mass
r	Related to the reduced transformed quantities
s	Related to the transformed quantities

Subscripts

d	Related to detent forces
em	Related to electromagnetic forces
ext	Related to external forces
k	Related to k-th winding
l	Related to l-th winding
$p-1$	Linked to the $(p-1)$-th harmonic of the magnetic flux linkage
$p+1$	Linked to the $(p+1)$-th harmonic of the magnetic flux linkage
q	Related to the radial motion
x	Related to the X-axis motion
y	Related to the Y-axis motion
z	Related to the axial motion
β	Related to the angular motion
γ	Related to the angular motion around the Y-axis
λ	Related to the angular motion around the X-axis
∞	Related to an infinite spin speed

Greek characters

α	Rotor spin position with respect to the stator
β	Rotor angular displacements in complex coordinates
β	Rotor angular displacements in matrix form
γ	Rotor angular displacement around the Y-axis
δ	Angular position of the phase windings
ϵ	Rotor radial displacement
λ	Rotor angular displacement around the X-axis
ν	Whirl angle
σ	Real part of the eigenvalues
ϕ	Magnetic flux linkage
$\mathbf{\Phi}$	Magnetic flux linkages
$\omega = \dot{\alpha}$	Rotor spin speed
ω_e	Electrical pulsation of the currents
ω_c	Rotor spin speed corresponding to the windings electrical pole
ω^*	Imaginary part of the eigenvalues

whose currents are controlled to generate an attractive force on a ferromagnetic rotor [2–4]. Although they present high stiffnesses and power densities, their use is still inadequate for some applications where compactness, reliability and low power consumption are required. By contrast, PMBs rely solely on passive phenomena. Among these, the bearings exclusively based on permanent magnets (PMs) achieve reasonable stiffnesses and high power efficiencies [5–9]. However, as stated in Earnshaw's theorem, an object can not be fully levitated through PMs alone [10]. Electrodynamic bearings (EDBs) also belongs to PMBs. Their operation depends on currents induced by the relative motion between the magnetic field produced by PMs and the conductors. Despite their low stiffnesses, these bearings are more compact, more reliable and less complex to implement. EDBs can be classified into two categories: radial and thrust bearings. Significant research has already been focused on the former, on both bearing topologies [11] and model [12–15] levels, notably highlighting critical stability issues. By contrast, electrodynamic thrust bearings (EDTBs) have yet to be investigated in such depth.

In the last forty years, three EDTBs topologies have mainly been studied [16–18]. On this basis, the range of topologies respecting the null-flux principle was subsequently generalised and thus extended in [19]. Despite this large set of possible designs, to date, almost all research efforts have been focused on the topology presented in [17]. This topology was notably successfully implemented and tested in fully passive magnetic suspensions [20–22]. Those experiments supported the derivation of models allowing us to analyse the axial stability and stiffness of these bearings.

Two models describing only the axial dynamics and quasi-static behaviour of EDTBs have been developed in the last few years. The former is a linear state-space representation whose scope is narrowed to the topology proposed in [17], an