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Control volume based modelling in one space dimension of oscillating, compressible flow in reciprocating machines

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Abstract

We present an approach for modelling unsteady, primarily one-dimensional, compressible flow. The conservation laws for mass, energy, and momentum are applied to a staggered mesh of control volumes and loss mechanisms are included directly as extra terms. Heat transfer, flow friction, and multidimensional effects are calculated using empirical correlations. Transformations of the conservation equations into new variables, artificial dissipation for dissipating acoustic phenomena, and an asymmetric interpolation method for minimising numerical diffusion and non physical temperature oscillations are presented. The capabilities of the approach are illustrated with an example solution and an experimental validation of a Stirling engine model.

Keywords: Oscillating compressible flow; One-dimensional models; Control volume; Numerical diffusion; Artificial dissipation

1. Introduction

Simulation and optimisation of many machines with reciprocating pistons, such as compressors, engines, heat pumps and coolers, are at a threshold. Many machines in use today have already been optimised using analytical models or classical numerical models. In these models significant simplifying assumptions are made about the fluid dynamics and thermodynamics of the working fluid in the machines in order to reduce the complexities of the models and the efforts required to obtain solutions. But the simplifying assumptions can limit the ability of the models to correctly predict machine performance and hence reduce the likelihood that the optimum design parameters for the machines can be found using the models. New modelling approaches with a more accurate prediction of machine performance are thus needed to further optimise the machines. The new modelling approaches have to remain practical with respect to the computer power required to perform design optimisation. In this paper we present a control volume based

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Nomenclature

$A_{\mathrm{c},i}$	cross section at centre of volume i [m^2]
$A_{\mathrm{c,ref},i}$	reference cross section in volume i [m^2]
$A_{\mathrm{ht},i,k}$	heat transfer area of surface segment k in volume i [m^2]
c_{v}	specific heat at constant volume [J/(kg K)]
$\Delta p_{\mathrm{f},i}$	frictional pressure loss in volume i [Pa]
$h_{\mathrm{conv},i,k}$	average convective heat transfer coefficient between gas and surface segment k in volume i [W/(m^2 K)]
$h_{\mathrm{gas},i}$	enthalpy at centre of volume i [J/kg]
E_i	total energy in control volume i [J]
$F_{\mathrm{AD},i}$	artificial dissipation force in volume i [N]
$F_{\mathrm{wall},i}$	wall friction force in volume i [N]
l_i	length of volume i [m]
$\tilde{\mu}_1$	artificial dissipation coefficient [s/m]
$\tilde{\mu}_2$	artificial dissipation coefficient [s/m]
m_i	mass in volume i [kg]
$\dot{m}_i$	mass flow rate at centre of volume i relative to volume i [kg/s]
p_i	pressure at centre of volume i [Pa]
p_{mean}	mean pressure in computational domain [Pa]
$\tilde{p}_i$	interpolated pressure at flow area change in volume i [Pa]
R	gas constant [J/(kg K)]
ρ_i	gas density at centre of volume i [kg/m^3]
u_i	internal energy per unit mass in volume i [J/kg]
U_i	internal energy in volume i [J]
v_i	specific volume at centre of volume i [m^3/kg]
V_i	size of volume i [m^3]
$\overline{V}_i$	velocity at centre of volume i relative to volume i [m/s]
$\overline{V}_{\mathrm{CS}}$	velocity of x-coordinate system relative to inertial coordinate system [m/s]
T_i	temperature at centre of volume i [K]
$T_{w,i,k}$	wall temperature of surface segment k in volume i [K]
x	axial space coordinate [m]

formulation suitable for making distributed models of unsteady compressible flow that is primarily one-dimensional. The method was developed for use in models of Stirling machines where it has been successfully applied [1–3].

We work with Stirling machines, and hence we present the modelling approach with Stirling engine simulation as background. However, the approach is not Stirling specific, and we believe it is applicable in other fields of engineering.

1.1. Related methods

The choice of modelling approach for compressible flow in a reciprocating machine depends on the type and geometry of the machine to be modelled and on the phenomena that need to be resolved. It is also important to remember that steady state solutions, that are periodic in nature due to the reciprocating piston movements in the machines, are usually needed for optimising the steady state performance of the machines. Steady state periodic solutions can be found either by successively simulating a sufficient number of revolutions or by using specialised numerical methods, such as shooting, collocation, or harmonic analysis, for solving boundary value problems; some methods, such as harmonic analysis, can find only steady state solutions. The computational expense of finding periodic steady state solutions depends on the model, numerical method, and