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Role of curvatures in determining the characteristics of a string vibrating against a doubly curved obstacle

Harkirat Singh *, Pankaj Wahi

Mechanics and Applied Mathematics Group, Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur 208016, India

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ABSTRACT

The motion of a string in the presence of a doubly curved obstacle is investigated. A mathematical model has been developed for a general shape of the obstacle. However, detailed analysis has been performed for a shape relevant to the Indian stringed musical instruments like *Tanpura* and *Sitar*. In particular, we explore the effect of obstacle's curvature in the plane perpendicular to the string axis on its motion. This geometrical feature of the obstacle introduces a coupling between motions in mutually perpendicular directions over and above the coupling due to the stretching nonlinearity. We find that only one planar motion is possible for our system. Small amplitude planar motions are stable to perturbations in the perpendicular direction resulting in non-whirling motions while large amplitude oscillations lead to whirling motions. The critical amplitude of oscillations, across which there is a transition in the qualitative behavior of the non-planar trajectories, is determined using Floquet theory. Our analysis reveals that a small obstacle curvature in a direction perpendicular to the string axis leads to a considerable reduction in the critical amplitudes required for initiation of whirling motions. Hence, this obstacle curvature has a destabilizing effect on the planar motions in contrast to the curvature along the string axis which stabilizes planar motions.

1. Introduction

In this paper, we revisit the problem of non-planar motions of a string vibrating against a boundary obstacle, which has been discussed in [1] and extend it to the case of a doubly curved obstacle which is encountered more frequently in real practical situations. For example, it is evident from Fig. 1 that the bridge over which the string passes in Indian stringed musical instruments like *Sitar* and *Tanpura* is curved both along the axis of the string as well as in the direction perpendicular to the string axis. Similarly in other practical situations like lifts, elevators and ropeways, the string passes over a pulley which has curvature along its cross-section. Fig. 1 shows the bridge of *Tanpura* in which the curvature perpendicular to the string axis is much more prominent than the curvature along the length of the string. A similar type of geometry is also observed in *Sitar* with the difference that in case of *Sitar*, the curvature of the bridge, especially the one along the direction perpendicular to the string axis, is relatively smaller than the curvature observed in *Tanpura*. In our previous study [1], it was shown that the geometry of the obstacle along the string axis is very critical while studying the coupling between

* Corresponding author.
E-mail address: harkirat236@gmail.com (H. Singh).

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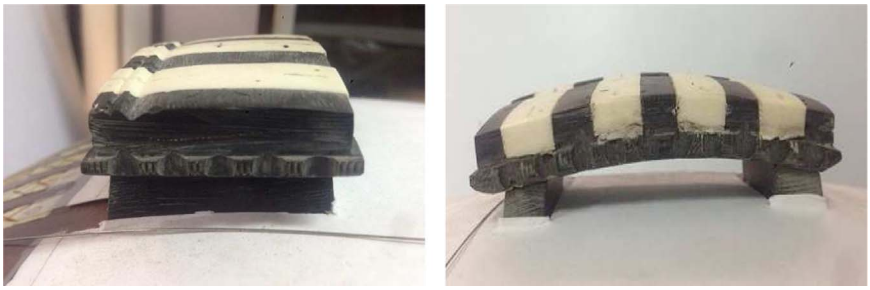

Fig. 1. Depiction of curvatures in the bridge of an Indian stringed musical instrument, *Tanpura*. Left: Front view (The length of the string is visible in this view). Right: Side view.

the motions in mutually perpendicular directions as well as the stability of planar motions. In this paper, we investigate the role of the obstacle geometry especially the curvature in the plane perpendicular to the string axis on the dynamics of the string.

Non-planar motions of strings have been subjected to numerous investigation in literature [2–11] where coupling between mutually perpendicular motions and stability of these nonlinear oscillations have been discussed in detail. However, all of them did not consider the obstacle which is relevant to the current study. The first study on non-planar vibrations of a string in the presence of an obstacle has been recently reported [1,12–14]. However, there are several studies dealing with planar vibrations of string in the presence of an obstacle [15–27]. Most of these studies did not focus on the complicated modal interaction between the in-plane modes because of the curved obstacle except for Mandal and Wahi [24–26]. The detailed study by Mandal and Wahi [24–26] clearly outlined the complex vibrational behavior that gets introduced for motions along the normal to the obstacle geometry. This analysis has been extended in [1] where the authors studied the non-planar motions of a string in the presence of a singly curved rigid obstacle considering both wrapping and stretching nonlinearities. The effect of the curvature on the number and stability of possible planar motions was studied therein. In this paper, this analysis has been extended to the case of a doubly curved obstacle which is a closer realization of a bridge used in the Indian stringed musical instruments.

The present study is organized as follows. In Section 2, we derive the equations of motion for a string vibrating against a doubly curved obstacle as nonlinear partial differential equations (PDEs) taking into account the stretching and wrapping nonlinearities. In the same section, we obtain reduced order equations in the form of nonlinear ordinary differential equations (ODEs) using Galerkin projections for the special case of an obstacle resembling the bridge of *Tanpura* and *Sitar*. The dynamics of the system is numerically investigated using these coupled nonlinear ODEs in Section 3. In the same section, we explore different types of non-planar motions and obtain the critical amplitude of the planar motion corresponding to transition between them. The effect of system parameters on this critical amplitude is presented as well. Finally, we conclude our study in Section 4.

2. Mathematical model

A schematic representation of the model under consideration is shown in Fig. 2. It comprises an ideal string (with no bending stiffness and internal damping) which can vibrate along the Y- and Z-axis. The string cannot penetrate the surface of the obstacle. Hence, we impose the constraint that there is a wrapped portion of the string which follows the geometry of the obstacle. The most general form for the obstacle geometry in the $X-Y-Z$ space can be written as

$$\zeta(X, Y, Z) = 0. \tag{1}$$

Since the obstacle poses constraint on the movement of the string in only one direction (perpendicular to itself), we can consider either the Y or the Z motion of the string to be unconstrained while the other one follows the constraint as per (1).

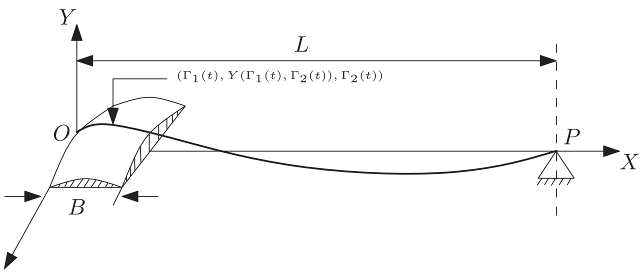

Fig. 2. Schematic representation of the string vibrations in musical instruments like *Tanpura* and *Sitar*.