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Transfer matrices to characterize linear and quadratic acoustic black holes in duct terminations

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ABSTRACT

The acoustic black hole (ABH) effect for sound reduction in duct terminations can be accomplished by means of retarding structures. The latter act as waveguides and their performance relies on two factors. First, a power-law decay of the duct radius and, second, an appropriate dependence of the wall admittance with the duct radius. In theory, the ABH can be achieved placing a set of rigid rings inside the duct, with inner radii and inter-spacing decreasing to zero as the tube end section is approached. In this work we focus on the linear and quadratic inner radius decay cases, referred to as the linear and quadratic ABHs. To begin with, analytical expressions are derived for the quadratic ABH and compared to those of the linear one. In both cases the solutions become singular at the final section of the duct. The wall admittance manifests the same behavior. Therefore, one has to deal with imperfect ABHs ending before the singularity, even in the best case scenario. Yet in practice, one may encounter further factors that deteriorate the ABH behavior. The number of rings and cavities between them is finite as it is the ring thicknesses. Damping also plays an important role. It is herein proposed to analyze the influence of all these factors on the reflection coefficient of the ABHs by means of the transfer matrix method (TMM). Transfer matrices are presented which allow one to relate the acoustic pressure and acoustic particle velocity between different sections of the retarding structure. They constitute a quick and valuable tool for an initial design of ABHs.

1. Introduction

The acoustic black hole (ABH) effect is a passive approach for the control of vibrations and sound (see e.g., the recent review in [1]). The effect is achieved by means of a retarding structure (or a tailored geometry). Such a structure induces a power-law-like decay of the velocity of the waves that propagate in it. In theory, an incident wave will become trapped and never reach the edge of the structure. The propagation wave velocity tends to zero with distance in such a way that a wave would spend an infinite amount of time to get to the boundary. Therefore, no reflection can occur from there which motivates the term “acoustic black hole”.

To date, most efforts have been placed on ABHs for flexural waves in beams and plates. One-dimensional ABHs can be achieved by varying the beam and/or plate edge thickness following a power law profile. The idea was first proposed in [2] and became more popular with the works in [3–5]. In practice, a perfect ABH effect cannot be manufactured because it

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would require a wedge of infinite extent. However, as quoted in [3,4,6], one can partially compensate for the imperfection of a finite thickness edge by covering the tip of the profile with a damping layer. Yet this is not the sole type of imperfection one may encounter in real implementations of ABHs. For instance, the effects of deformations due to wedge attachments to plates [7], or due to stress relaxation when manufacturing the ABHs [8], are of practical importance and have been recently analyzed in the referenced works. The ABHs have been proved to be quite robust to these imperfections, still providing low reflection coefficients. On the other hand, it is worthwhile mentioning that two-dimensional ABHs have also been designed. These usually consist of plates with tapered holes having an appropriate inner profile [9–11].

As opposed to ABHs for beams and plates, less attention has been paid to the possibility of using the acoustic black hole effect for sound reduction. The first proposal in that direction was presented in [12]. A theoretical analysis was made for a retarding structure consisting of a set of rings and cavities placed at the termination of a duct. The approximated wall admittance of that structure was such that when combined with a power-law decaying radius for the inner rings resulted in an ABH effect, with no sound waves reflecting from the end of the duct. Practical realizations of the waveguide in [12] for linear and quadratic decaying ring radii have been recently attempted in [13,14]. Besides, other possibilities for one-dimensional sound reducing ABHs are suggested in [1]. Two-dimensional designs of ABH sound absorbers have been also lately proposed in analogy to optical black holes. These usually consist of a graded index metamaterial shell that acts as an impedance matching layer between air and an inner porous absorber core [15,16], or second metamaterial [17], which dissipates acoustic energy. This results in an omnidirectional broadband absorbing device.

This paper focuses on the ABH for ducts suggested in [12], which can be seen as a particular case of a reactive anechoic termination. Anechoic terminations are of practical importance in the design of wind tunnels, fan and propeller test rigs, and for the acoustic characterization of mufflers. The measurement of mufflers' transmission loss, for instance, usually relies on the so-called two-microphone transfer function method [18], or some of its enhanced variations [19,20]. The corresponding setup requires a duct with an anechoic termination. Research on them have been going on for decades and very different approaches have been attempted, which comprise from using wedges [21–23], absorbing layers [24,25], conical, exponential and catenoidal profiles for smoothing the impedance mismatch [26,27], using micro-perforated panels [28,29] or resorting to active noise control strategies [30]. The ABH may be viewed as an alternative strategy to achieve anechoicity.

In this work we are interested in the particular cases of linear and quadratic decaying ring radii for the retarding structures in [12], which will be hereafter referred to as the linear ABH and the quadratic ABH, following the nomenclature in [13,14]. To begin with, we will develop analytical expressions for the quadratic ABH to complement those for the linear one in [12]. It turns out that the acoustic pressure of the linear and quadratic ABHs becomes singular at the duct end section. The same happens to the wall admittance. As a consequence, even for theoretical computations of the waveguide reflection coefficient, one has to assume an imperfect ABH that finishes before the end section of the duct. Yet, when building an ABH (see [13,14]) that is not the sole source of limitations. It is also necessary to deal with a finite number of cavities, separated by rings of finite thickness, which follow a certain spatial distribution. The admittance of such a realistic waveguide departs from the theoretical one and thus restrains the ABH effect. The situation is somewhat akin to that encountered in structural ABHs, where the potential decaying wedges should have infinite extent but have to be truncated for practical implementations.

A precise analysis of the linear and quadratic ABHs could be carried out by means of finite element simulations (FEM). However, FEM can be time consuming and costly if one has to check a large number of configurations for varying parameters. Therefore, prior to FEM it is herein proposed to make use of the transfer matrix method (TMM, see e.g., [31]) for acoustic filters. The ABH retarding structure is approximated as a duct of varying cross-section with several branch cavities appended to it. The relation between state vectors of acoustic pressure and acoustic particle velocity at different sections of the waveguide is then established by means of analytical transfer matrices. The TMM enables one to quickly analyze the parameters influencing the performance of the retarding structure, such as the number and size of rings and cavities, their location, the damping, the ring thicknesses, etc. Once the dependence of the ABHs on those parameters is made clear, one could chose a small set of the most promising configurations for a more detailed FEM analysis (FEM simulations are, however, out of the scope of this paper and will be carried out in future work). The TMM has been traditionally applied in many areas of acoustics. To mention a few, these comprise from the design of mufflers [31], to that of musical instruments [32,33] or to articulatory speech synthesis [34].

The paper is organized as follows. Section 2 first presents the governing equation of the ABH. Then, the theory behind the linear ABH in [12] is reviewed for completeness (though presented in a slightly different way) and analogous expressions get derived for the quadratic ABH. In Section 3, transfer matrices are constructed and combined to deal with realistic exemplifications of the linear and quadratic ABH retarding structures. Comparisons between the theoretical and realistic ABHs are presented in Section 4, together with the dependence on several design parameters. The conclusions close the paper in Section 5.

2. The acoustic black hole effect in duct terminations

2.1. Governing equation

The equation that governs plane wave propagation in an axisymmetric waveguide of varying cross section $S(x)$ and wall admittance $Y(x)$ can be determined from the linearized continuity and momentum conservation equations. The result is a

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