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Neutral inclusions for diffusive acoustic fields

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ABSTRACT

We predict scattering cancellation in diffusive transport of acoustics waves propagating through multiple scattering media in the stationary limit. This would enable sensing of diffusive sound without disrupting the exterior acoustic field. We present design schemes for making spherical or cylindrical core-shell structures with multiple layers, characterized by homogenous and isotropic diffusion coefficients, neutral to an arbitrary applied multipole field. The double-layered sphere is found to support transparency to two concurrent multipole fields and unique cloaking solutions of arbitrary multipole order. One extra degree of freedom is provided by every layer added to the core-shell structure which may be exploited with our iterative formula for effective diffusivity for cloaking of additional field terms. From this we pass over to the long wavelength limit of ballistic sound and provide formulas for effective mass densities of multi-layered structures in spherical and cylindrical geometries with respect to multipole pressure fields.

1. Introduction

An acoustic cloak is a shell surrounding a target so that sound incident from any direction passes through and around the cloak, making the cloak and the object undetectable. One cloaking approach is to develop and use metamaterials for guiding sound around the target. This approach prevents wave interaction with the object, and was originally described for electromagnetic waves [1,2], and subsequently extended to acoustic [3–5] and elastic waves [6,7], and also to mass diffusion [8]. Another route to cloaking is to also recognize the object properties and construct the cloak to cancel the dominant scattering, rendering the cloak and the object nearly invisible [9]. Using a properly designed isotropic material shell one can achieve cancellation of the monopole and dipole scattered terms. This amounts to making the shell and the target have effective density and compressibility of the surrounding medium [10,11], so that they behave as a neutral acoustic inclusion [12,13]. Acoustic scattering cancellation has recent been experimental realized in cylindrical geometry by matching the effective density and compressibility to a water background [14].

In many situations, sound propagation is not ballistic and cannot be described adequately by the acoustic wave equation. For instance, in bubbly water or in other systems containing many randomly distributed scatterers, sound can be thought of as performing a random walk. Effectively, this random walk slows the wave propagation down and scrambles the phase leading to diffusive transport [15]. We note that the common approach to describe acoustic wave propagation in bubbly media is to use an effective medium theory that provides an effective phase speed and attenuation while taking into account the void fraction and bubble dynamics [16]. However, if the transport mean free path length in the disordered scattering medium is shorter than the relevant system size and larger than the wavelength of sound, multiple sound scattering can be

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described by diffusion theory and it is meaningful to talk about a diffusion coefficient.

In this paper, we seek neutrality (or transparency) to multipole fluxes in the stationary limit of diffusive acoustics. This would enable a sensor to detect diffusive sound without disrupting the exterior acoustic field in analogy with the ballistic case [17]. The experimentally determined quantity is the transmitted flux (i.e. the acoustic intensity which is proportional to the square of the detected pressure) which is related to the energy density via Fick's law [18]. In steady-state transport measurements the outcome is determined by the transport mean free path, whereas the diffusion coefficient is obtained from dynamic experiments. In contrast to the acoustic wave equation [4] the time-dependent diffusion equation [8] is not form invariant under a general coordinate transformation so that a diffusive acoustic cloak cannot be designed using a spatial transformation. For stationary diffusion the transformational design approach does apply and gives a spatially dependent diffusivity tensor, which can be difficult to realize in practice. An acoustic bilaminate shell was designed to cancel the scattering of monopole, dipole and quadrupole modes of a compressional (ballistic) wave at the same frequency [19] and there is nothing to prevent this from being achieved also in the diffusive limit. A scheme for calculating the partial scattering amplitudes and the related scattering phases for an arbitrary layered distribution of acoustic material properties was presented in Ref. [20], where the possibility of reducing the acoustic radiation force, which they referred to as suppressing the transport cross section, was investigated by numerical optimization. Ideally, setting all the scattering phases equal to zero yields zero transport cross section with cloaking as a result. However, in the diffusive limit the concept of phase and coherence of a wave is lost altogether.

We present a scheme for making spherical core–shell structures with multiple layers, characterized by isotropic and homogeneous diffusion coefficients, neutral to arbitrary multipole fields applied at infinity. The double-layer shell is found to support transparency to two concurrent multipole fields and unique cloaking solutions of arbitrary multipole order $n \geq 1$. This is an extension of dipole ($n=1$) cloaks appearing in other contexts [21–25]. The number of simultaneous transparency solutions, i.e., the two multipole diffusive fields that can penetrate the core without leaving a scattered field outside the shell, is consistent with the result of scattering cancellation of ballistic waves by an acoustic bilaminate shell [19]. We note that thermal transparency and invisibility has been predicted using the scattering cancellation and mantel cloaking concepts for heat diffusion [26]. In this context, our result applies to stationary heat diffusion and constitutes an extension of the dipole heat flux and single-layer shell considered in Ref. [26], which does not support cloaking solutions. One extra degree of freedom is provided by every layer added to the core-shell structure which may be exploited with our iterative formula for effective diffusivity to obtain neutrality to additional field terms. As an example, we give the design recipe for a three-layer shell and show that it supports simultaneous cloaking solutions of two different multipole fields. These results can be adopted to physical phenomena described by the Laplace's equation in spherical symmetry and cylindrical symmetry may be treated with the same methodology. Finally, we pass over to the long wavelength limit of ballistic sound and provide formulas for effective mass densities of multi-layered structures in spherical and cylindrical geometries with respect to multipole pressure fields applied at infinity.

2. The diffusion approximation

The description of acoustic waves propagating through strongly scattering media is facilitated by the use of the diffusion approximation, in which all phase information is neglected and the propagation of the multiply scattered waves is treated as a random walk process, see Fig. 1(a). Within this approximation, the dynamic transport is described in terms of the wave diffusion coefficient $D = v_E l_t/3$, where v_E is the energy velocity, which corresponds to the average local velocity of energy transport in the diffusion process, and l_t is the transport mean free path, or the distance the waves must propagate until

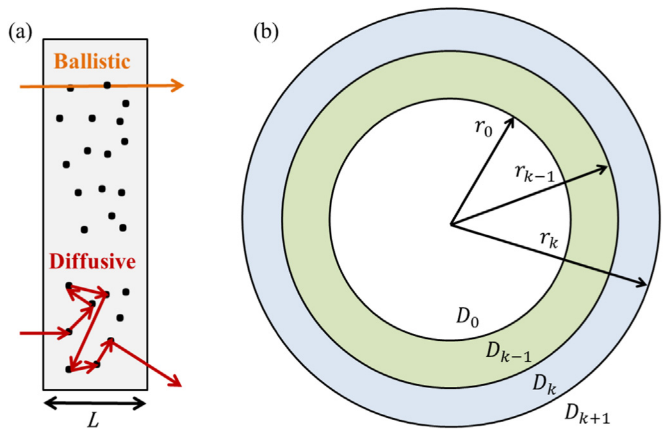

Fig. 1. (a) Illustration of ballistic versus diffusive sound paths in a scattering medium of length L. (b) Cross-section of a spherical core with diffusivity D_0 and radius r_0 coated by k concentric laminated layers with diffusion coefficients $D_1, \ldots, D_k$ and radii $r_1, \ldots, r_k$ immersed in a medium with diffusivity D_{k+1}.