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Characterizing the robustness and susceptibility of steady-state dynamics in post-buckled structures to stochastic perturbations

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ABSTRACT

Recent reports exemplify both the potential and the concerns encountered in implementing post-buckled structures in diverse applications. By numerical and experimental methods, characterizations have provided useful insights on the dynamic sensitivities of such bi- or multistable structures subjected to either harmonic or stochastic excitations. Yet, the more realistic scenario of combined harmonic and stochastic loading has been not been closely examined so that such sensitivities might be fully illuminated. To provide a more complete understanding on the robustness and susceptibility of multistable structures to dynamic state transitions, this research establishes new analytical and experimental methods to quantify the likelihood of triggering transitions among dynamic regimes of an archetypal post-buckled structure as a result of combined harmonic and stochastic loading. It is discovered that persistent periodic snap-through dynamics are rapidly disabled by additional noise excitation when the harmonic excitation contribution occurs at frequencies close to the linearized resonance. The extra noise may also drastically compromise the integrity of small-amplitude periodic responses that occur at frequencies around one-half of the linearized resonance. Particular relative proportions of the noise standard deviation and harmonic excitation amplitude are uncovered that most readily compromise the robustness of a given steady-state dynamic regime. The analytical method also complements prior developments focused on stochastic resonance by uncovering the broader perspective of dynamic sensitivities of post-buckled structures under arbitrary combinations of harmonic and random driving loads.

1. Introduction

Snap-through is the dynamic response associated with displacements from one stable equilibrium to another in a bistable, multistable, or post-buckled structure, and it results in a large sudden conversion between potential and kinetic energy that is dissipated in time by damping effects [1]. This unique, nonlinear dynamic behavior is important for a variety of engineering applications. Persistent snap-through oscillations are favorable for harvesting ambient vibration energies as a sustainable electrical power resource using bistable electromechanical devices. Namely, when the exciting energies are insufficient to greatly drive a linear energy harvesting platform, the same inputs may induce stochastic resonance-like

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behaviors in bistable systems, promoting energetic oscillations and considerable electric power production [2–4]. Transitions among post-buckled states also facilitates novel, adaptive capabilities for engineered structures, including tunable frequency sensitivities [5], graded energy absorption features [6–8], exceptional actuation authority [9], and large multistable shape morphing [10]. Yet, considered from the lens of different applications, snap-through can be detrimental to the integrity and performance of critical structural systems. For example, aircraft may be subjected to skin buckling due to combined thermal and mechanical loading on the thin surface panels and control surfaces [11]. Such bi- or multistability could rapidly compromise system integrity if repeated dynamic transitions among the stable equilibria are activated [12,13]. Thus, archetypal, slender aircraft components have earned a recent focus towards developing predictive tools that inform deployment of such structural elements [14,15]. Indeed, wherever slender structural constituents are employed in engineering systems undergoing compressive loads there is the possibility that buckling may be induced at one or more locations, which poses a genuine concern for triggering snap-through events as a result of additional lateral stresses or loads [16].

Among these diverse investigations, the dynamics of *bistable structures* have been the focus of numerous investigations. The existence of two stable equilibria is amenable for *analytical methods* that yield general insights, albeit at a resolution viable for general engineering design guidance rather than exacting precision. Because more complex multistable structures do not involve a considerable change in the global dynamic behaviors that may result [17], bistable structures are a suitable substitute to investigate the fundamental dynamic phenomena and sensitivities of a whole class of engineering systems: structures with more than one statically stable equilibria.

As a result, a broad range of analytical studies have been undertaken on the structural dynamics of post-buckled structures and bistable oscillators to obtain important insights. Moon and Holmes [18] investigated an archetypal bistable cantilever that was buckled using magnetic forces, and employed analytical tools including Poincaré sections and Melnikov theory to yield important knowledge on the persistence of harmonically-excited steady-state motions. Later, Szemplińska-Stupnicka and Rudowski [19] utilized approximate analytical solutions to the governing equations via assumed sinusoidal response behavior to predict the onset of dynamic transitions in harmonically-excited bistable structures and the corresponding spectral energy distributions. Virgin [20] characterized the frequency sensitivities of archetypal mechanical and structural systems due to load and geometric constraints that could promote post-buckling, revealing that buckling is easily identified by the vanishing of the fundamental natural frequency. Recently, the idea of using bistable structures for vibration energy harvesting applications gave new motivation for analytical study that could uncover important knowledge for the harvester's best implementation. Thus, researchers have developed analyses that help identify methods for harnessing bistable energy harvesters in applications with harmonic [21,22] or stochastic exciting vibrations [23] that may be favorably converted into electric power. In an adaptive structure application, Kidambi et al. [7] used an analytical approach based on the harmonic balance method to elucidate the roles of symmetry, frequency, and excitation amplitude on the damping adaptation achieved by embedding a bistable constituent in an elastic mechanical system. Continued investigations are identifying the sensitivities of bistable structures under transient [24] or impulsive [25] conditions, which furthers the knowledgebase for effective utilization or operation of bistable engineering systems in a diverse range of applications. While this brief review is by no means comprehensive, the breadth of knowledge derived from *analytical studies on bistable structural dynamics* – particularly those incorporating supporting numerical or experimental evidence – has shed remarkable light and understanding on the dynamic behaviors unique to bistable systems but also manifest in the more complex multistable structural systems.

On the other hand, there is an important omission from this portfolio of knowledge. Although many scenarios involve harmonic or stochastic energies which drive or excite a post-buckled structure, it is rare that purely tonal or purely stochastic force/stress exists in the natural or engineered world. For instance, it is common that a background level of noise is not significantly less than spectral amplitude peaks associated with a mechanical or structural vibration response from harmonic inputs [26,27]. Considering the alternative perspective, even near-random excitations such as turbulent boundary layers past structural surfaces can induce vortex shedding that results in a periodic contribution to the total structural load [28]. More generally, some engineering systems, in particular aircraft [29,30], undergo an exhaustive variety of loading over the course of typical operations such that the structural components are at some times subjected to strongly tonal, mostly stochastic, or an even mixture of harmonic and stochastic excitations. While previous studies have analytically or numerically investigated the sensitivities of monostable nonlinear oscillators to such combined harmonic and stochastic excitations [31–35], there is a lack of comparative, analytically-based understanding for structures that possess more than one stable equilibrium. Thus, due to the myriad examples of post-buckled structures in modern and emerging engineering applications and the broad relevance of combined harmonic and stochastic inputs, the motivation of this research is to bridge the knowledge gap and lay a foundation for the study of harmonically- and stochastically-excited bistable structures using *new analytical* and *experimental methods* that yield conclusive insights.

The following section describes the model of the archetypal post-buckled structure examined in this work and the analytical framework employed to pursue understanding of the system behaviors in consequence to a combination of harmonic and stochastic excitation. The experimental system is then introduced and experimental results are compared to analytical predictions to correlate the conclusions from experiments to those suggested by the analysis. Concluding remarks summarize the principal findings of this research.

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