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The acoustic Green's function for swirling flow in a lined duct

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ABSTRACT

This paper considers the acoustic field inside an annular duct carrying mean axial and swirling flow and with either acoustically hard or lined walls. The particular aim is to compute the Green's function, which is required for predicting the noise generated by known acoustic sources, both approximately and numerically. Asymptotic approximations for first the eigenmodes and then the Green's function are derived in the realistic limit of high reduced frequency, which are found to agree very favourably with results determined numerically, even for relatively modest frequencies. Using a blend of uniform WKB asymptotics and numerics, singular cases in which multiple turning points are present are treated, allowing computation of accurate results across a very wide range of parameter values and flows.

1. Introduction

The propagation of acoustic waves through swirling annular duct flow is a crucial issue for the understanding and prediction of sound levels produced by aircraft engines, and therefore for the control of noise pollution around the world's busiest airports. A number of the key technological issues and a discussion of the most important turbomachinery-based noise sources are given in [22].

Noise prediction is often completed using an acoustic analogy. Lighthill [14] famously derived the first acoustic analogy by rearranging the Navier–Stokes equations into a single equation, with the left-hand side being the wave operator in quiescent fluid acting on the density perturbation and the right-hand side being thought of as the sound sources. A formal solution of this equation, assuming that the right hand side is known, is computed as a convolution of the source terms and the Green's function, which in this case is the simple free-space Green's function of the wave operator (see for example Duffy [7]). Lighthill's analogy has been extended in a number of ways, which include considering moving surfaces in the flow, Curle [5] and Ffowcs Williams and Hawkings [8], and choosing a different dependent variable on the left-hand side (for instance, Goldstein [9] and Morfey and Wright [19]). Furthermore, a number of authors have recast Lighthill's analogy to explicitly account for non-trivial base flows by changing the operator on the left-hand side. Lilley [15] extended Lighthill's analogy into a form more suitable for modelling high-speed jet noise by introducing a third-order operator on the left-hand side to represent unidirectional base shear flow. This is often approximated by the linear Pridmore-Brown operator [9], acting on the logarithm of the pressure. In a different direction, Posson and Peake [25] considered an axially sheared and swirling base flow in a duct, and rearranged the governing equations into the form of a (this time) sixth-order linear operator acting on the pressure perturbation. In order to solve this version of the acoustic analogy, the Green's function in ducted swirling flow is therefore clearly required.

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It is important to choose a Green's function which is tailored to the geometry, thereby reducing the number of surface terms which need to be calculated in the convolution with the source terms. For example, for flow in an annular duct the Green's function should have appropriate boundary conditions on the inner and outer walls. A key first step is then to determine the allowed eigenmodes of the system. The eigenmodes in ducted swirling flow have been considered numerically by a number of authors, including early work by Posson and Peake [10] and Tam and Auriault [32]. In Cooper and Peake [4] and Heaton and Peake [11] these eigenmodes were calculated asymptotically in the limit of high frequency, using the WKB method. In Vilenski and Rienstra [34,33] a lined infinite duct with radially-sheared axial mean flow but with zero swirl is considered, leading to determination of the eigenmodes of the Pridmore–Brown [26] differential equation; both numerical and asymptotic results are presented, with particular focus on the trajectories of the eigenvalues as the wall impedance is varied. In Posson and Peake [23] and Posson and Peake [25] the sixth-order acoustic analogy was derived for a homentropic fluid, and both the eigenmodes and the Green's functions for a hard-walled duct were calculated numerically. A base flow with shear and swirl was considered in a hard-walled infinite duct. In Posson and Peake [24] the results were extended to a duct with acoustically lined walls. In this paper our aim is to extend the above work by finding the Green's function for the acoustic analogy derived in Posson and Peake [25], now using high-frequency asymptotics which we will compare with numerical results.

As we have already noted, the ducted swirling-flow Green's function is required for predicting the noise generated by known source distributions, as performed by Masson et al. [23], Posson and Peake [25,16]. Alternatively, the Green's function can be used in beamforming to infer information about noise sources and the effectiveness of the lining from acoustic measurements. Significant progress has been made, such as by Sijtsma [30], and beamforming is now one of the major processing tools used to analyse microphone array data in aeroengine noise tests. Inference of source information from far-field data of course requires knowledge of the propagation path from source to observer, which is why the Green's function is required; in practise only relatively simple Green's functions have been used to date, with the most complicated case only assuming radial, piecewise constant axial shear flow in the duct [31]. However, in reality the effects of the swirling flow are significant, and failure to include swirl in the Green's function can potentially lead to spurious source localisation. Application of results from the present paper to beaming in rotor-stator noise tests is therefore a promising line of further inquiry.

This paper is laid out as follows. In Section 2 we present the governing equations for the Green's function together with the boundary conditions in a lined duct, and discuss the swirling base flow. In Section 3 we discuss how to calculate the eigenmodes and the Green's function asymptotically in the high-frequency limit, with the numerical methods described in the Appendix. We analyse the results in Section 4, comparing our asymptotics with numerical results; we will see that the asymptotic eigenmodes and the Green's function are extremely accurate compared to the numerical results, even for very modest frequencies.

2. Acoustic analogy in swirling flow

We will model the aeroengine as an infinite cylindrical duct, although approximations with slowly varying axial ducts have previously been considered by Rienstra [27] and Cooper and Peake [3]. Let the inner and outer duct walls be given by $r^\ddagger = h^\ddagger$ and $r^\ddagger = d^\ddagger$ respectively, where the double dagger ‡ represents dimensioned coordinates. We non-dimensionalise all distances by $d^\ddagger$, so that the inner wall lies at $r = h := h^\ddagger / d^\ddagger$ and the outer wall at $r = 1$.

In Fig. 1 we see the cylindrical coordinate system, with x the axial coordinate, r the radial coordinate and θ the azimuthal coordinate. We let $\underline{u}$, $\underline{v}$ and $\underline{w}$ be the velocities in the x, r and θ directions respectively. We split the inviscid total flow (underlined) into a base flow (subscript 0) plus some small time-harmonic perturbations, so we have

$$(\underline{u}, \underline{v}, \underline{w}, \underline{\rho}, \underline{p}) = (u_0, v_0, w_0, \rho_0, p_0) + (u, v, w, \rho, p), \tag{1}$$

where $\underline{\boldsymbol{u}} = (\underline{u}, \underline{v}, \underline{w})$ is the total velocity of the air, $\underline{\rho}$ is the total density and $\underline{p}$ the total pressure. We non-dimensionalise all velocities by the speed of sound at the outer wall $r^\ddagger = d^\ddagger$, $c_0^\ddagger(d^\ddagger)$. Finally, we non-dimensionalise time by $d^\ddagger / c_0^\ddagger(d^\ddagger)$ and all frequencies by $c_0^\ddagger(d^\ddagger)/d^\ddagger$.

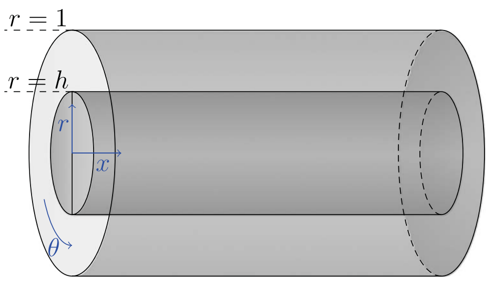

Fig. 1. Geometry of the duct.

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