Journal of Sound and Vibration 395 (2017) 317–327

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Splitting of quality factors for micro-ring with arbitrary point masses

Jung-Hwan Kim [a], Seok-Joo Kang [a], Ji-Hwan Kim [b,*]

[a] Department of Mechanical and Aerospace Engineering, Seoul National University, 1, Gwanak-ro, Gwanak-gu, Seoul 08826, South Korea
[b] Institute of Advanced Aerospace Technology, Department of Mechanical and Aerospace Engineering, Seoul National University, 1, Gwanak-ro, Gwanak-gu, Seoul 08826, South Korea

ARTICLE INFO

Article history:
Received 22 October 2016
Received in revised form
2 February 2017
Accepted 11 February 2017
Handling Editor: L.G. Tham
Available online 16 February 2017

Keywords:
Quality factor (Q-factor)
Micro-ring
Point mass
Imperfection

ABSTRACT

A ring resonator model with mass imperfections can be represented by a circular ring with irregularly distributed point masses. In this paper, in-plane vibration of the micro-ring is to be investigated. Also, the ThermoElastic Damping (TED) effect is considered in the structures. In order to get the temperature profile for the thermal flow, one-dimensional heat conduction equation is employed. Using the assumed mode shapes for the inextensional vibration of the imperfect micro-ring with TED, analytical expressions for the natural frequencies with concentrated masses are determined by the equations of motion. Then, the split of the natural frequencies are determined according to the effect of distributed point masses. Finally, imperfect micro-ring with TED is presented with the variations of Quality-factors (Q-factors) for lower and higher modes. Based on the verification of the present work, more parametric studies are performed for the ring with mass imperfections.

1. Introduction

Micro-Electro-Mechanical Systems (MEMS) and Nano-Electro-Mechanical Systems (NEMS) have been developed for rate sensors in sensitive devices such as resonators, *etc*. In this regards, the thermo-elastic damping (TED) is well known as the most important energy loss factors from internal friction for designing micro-and nano-structures. As a typical model for advanced structures, a ring model has been widely used as a part of various equipment. Zener [1] investigated local fluctuations within a vibrating solid structure due to internal friction by heat conduction effects. Lifshitz and Roukes [2] examined TED as a dissipation mechanism in micro and nano mechanical systems. Wong et al. [3] applied Zener's theory to a thin circular silicon ring structure, and expressed the Q-factor by considering the in-plane flexural modes of vibration. Guo [4] studied thermo-elastic dissipation of micro-beam resonators with the generalized elasticity theory. Vahdat and Rezazadeh [5] investigated TED in a micro-beam resonator with axial and residual stresses by energy method. Joshi et al. [6] reviewed the relationships between dissipation and structural dynamics for systems with small linear damping in micro-mechanical resonators. Tai and Li [7] presented analytical model for TED in micro-mechanical resonators based on the heat conduction effect. Kim et al. [8] investigated the Q-factor relevant to TED in a rotating thin ring on the in-plane vibration. Salajeghe et al. [9] showed a non-linear analysis of TED on transverse motion of a thin circular plate using equations of

---

[*] Corresponding author.
E-mail address: jwhkim@snu.ac.kr (J.-H. Kim).

http://dx.doi.org/10.1016/j.jsv.2017.02.030
0022-460X/© 2017 Published by Elsevier Ltd.

motion. Moosapour et al. [10] studied TED in an atomic force microscopy resonator beam undergoing flexural vibration. Meanwhile, Tao et al. [11] experimentally explored the possibility of developing a novel micro-ring vibratory gyroscope based on a piezoelectric beam element. Also, Tallur et al. [12] presented a comparison of frequency scaling of quality factors of radial modes in a ring resonator. Wang et al. [13] analyzed a hermetically encapsulated breathe-mode ring resonator with a very high Q-factor through experiments. On the other hand, Peng et al. [14] showed a non-linear dynamic analysis of a micro-actuator based on the Euler-Bernoulli beam theory without considering TED. As in these works, micro-ring structures have been investigated for the perfect ring system. Lin [15] investigated the forced vibration of a beam subjected to a harmonic external force with the viscous and thermoelastic damping. Tunvir et al. [16] studied the thermoelastic dissipation of micro/nano beams of arbitrary rectangular cross-section with accurate satisfaction of the surface thermal condition.

Based on a practical point of view, structural imperfections are inevitable. That is to say, the irregularity of mass distribution makes crude accuracy of the device. In this regard, numerous researchers mainly investigated imperfect structures without considering the intrinsic damping mechanism for micro- and nano-structures. Previously, imperfect circular rings have been modeled without considering the TED mechanism. Fox [17] compared experimental data and analytical results to investigate the vibration of circular rings with discretely attached masses and springs. On the other hand, Hong and Lee [18] researched the distribution of finite mass and stiffness for the in-plane vibration of the ring, and compared with experimental results. Also, Laplace Transform was applied to analyze the deviations from the perfect model. Bisegna and Caruso [19] investigated using the perturbation expansion for the frequency split of a thin ring with small imperfections. Additionally, more works on the TED effect have been treated using a beam element. Li and Hu [20] presented micro-resonators with proof mass and the results were similarly applied to Lifshitz's thermo-elastic model. Also, Guo and Ti [21] practically applied MEMS beam resonators with high Q-factors to compensate for TED by considering the piezo-resistive effect. Kim and Kim [22] studied thermoelastic dissipation of the micro-ring model with discontinuous deviations.

In this paper, inextensional model of an imperfect micro-ring is investigated with the TED effect. Thus, heat conduction effect is considered, and frequency split due to the random point masses are analyzed. To determine the frequency split of the model, the equations of motion are adopted to obtain analytic solutions. As the validity of the present work, the results for the isothermal ring model with imperfections are compared with previous experimental data. Also, Q-factors are presented for various parameters of the micro imperfect ring model with TED.

2. Formulation

Fig. 1 shows a micro-ring with point masses, where a,b and d denote mean radius, radial thickness, and the longitudinal height of the ring, respectively. In addition, x, y and z are the local coordinates of the section of the ring model. Also, local displacement components u and v stand for the radial and tangential displacement components, while, θ_i denotes the angular location of attached i-th imperfect masses. In-plane flexural vibrations of the ring are described in terms of the displacements of the mid-point of the ring cross-section at the circumferential location θ as shown in the Figure.

2.1. Ring model with point masses

To analyze the imperfect micro ring model, Euler-Bernoulli assumptions are adopted, including the TED effect. That is to say, the model is thin enough as compared to the radius of the structure. Due to inextensional assumption of the ring, the strain-stress relationship in the radial direction and membrane strains are negligible [23].

Thus, bending strain with thermal expansion is expressed as in Ref. [24]:

$$\varepsilon_\theta = \frac{1}{E}\sigma_\theta + \alpha T \tag{1}$$

Then, the circumferential stress from the rearrangement of Eq. (1) is

$$\sigma_\theta = E(\varepsilon_\theta - \alpha T) \tag{2}$$

where ε_θ, E, α and T are circumferential strain, isothermal Young's modulus, thermal expansion coefficient, and reference temperature, respectively.

Also, the displacements of harmonic vibration with frequency ω in the relevant radial and tangential modes [19] can be

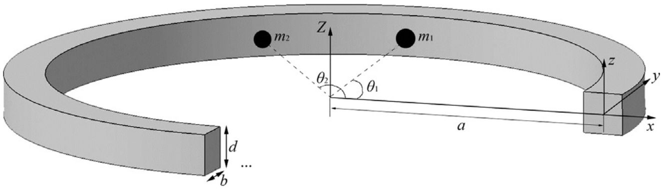

Fig. 1. Cross-section of a ring with point masses in global and local coordinate systems.