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Vibration confinement in beams

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ABSTRACT

We develop approaches for vibration confinement to within an arbitrary region in both non-fluid-loaded beams and fluid-loaded beams. The approach for non-fluid-loaded beams makes use of a novel forward/backward substitution algorithm that generates an evanescent response that exhibits exponential growth from one end of the beam to a specified node, and then exponential decay from that node to the other end of the beam. A weighted sum of such solutions supports vibration confinement while requiring only one actuator at each edge of the confined region. We also demonstrate vibration confinement with just a single actuator term when one edge of the confined region extends to the end of the beam. For fluid-loaded beams, we use a weighted sum of solutions having compact support to achieve near-confinement. While the displacement vector is nonzero outside of the confined region (as a consequence of the acoustic loading on the beam), its amplitude is significantly reduced.

1. Introduction

The control of structural vibrations and acoustic radiation has been an active research topic for over three decades. Many aspects of this ongoing research have been extensively documented in reference books [1,2]. In particular, there has been a considerable amount of work focusing on vibration confinement in flexible structures [3–9]. For example, Shelley and Clark [6] were able to localize all modes in a tridiagonal system with two control sensor/actuator pairs. However, the required number of pairs increases with the system bandwidth (i.e., the number of diagonals in the coefficient matrix). Other work [7] has either required a number of actuators that is equal to the dimension of the system of equations, or more recently, a limited number of actuators [8,9]. In the present paper, we focus on vibration confinement in beams. When there is negligible fluid loading (i.e., for beams in air), we use a finite difference approach and a novel solution algorithm that generates a response that involves exponential growth from one end of the beam to a specified node and exponential decay from that node to the other end of the beam. Furthermore, this response is associated with only three nonzero load terms, i.e., one at the specified node, and one at either adjacent node. We show that a weighted sum of such solutions (centered at different nodes) leads to vibration confinement to within an arbitrary region on the beam with only two actuators, i.e., one actuator at each end of the confined region. Outside the confining actuators, the vibration amplitude decays exponentially. We further show that only one actuator is required for vibration confinement when the region extends to one end of the beam.

Our approach is a result of research into alternative solution approaches for banded linear systems [10–13]. One form involves a forward substitution approach that is initiated by assuming the values for the first q unknowns (where q is the

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number of superdiagonals and $2q + 1$ is the system bandwidth). Every subsequent equation can then be solved for an additional unknown using the highest diagonal as a divisor. Since the approach assumes the value of the first q unknowns, it requires q additional unknowns that, once solved (via a $q \times q$ system), leads to the solution of the entire system. In the process, it decomposes the original system into q subsystems that can be solved in parallel.

A second form involves both forward and backward substitution. This is necessary when the forward substitution process leads to a solution dominated by an exponential growth response, which prevents the solution approach in [10–12] from working. However, if the right hand side (RHS) vector has only a single nonzero term located at $k=m$, a modified approach will work when forward substitution is performed for $1 \le k \le m$ and backward substitution is performed for $n \ge k \ge m$, where n is the number of unknowns. The backward substitution process also leads to a solution that exhibits an exponential growth response. However, because it grows exponentially as the solution progresses in the backward direction, it is observed as exponential decay [12,13].

For a tridiagonal system, a single nonzero RHS term remains after completion of the forward and backward substitution processes. This term (and the solution vector) can then be appropriately scaled to obtain the required solution. For a pentadiagonal system (which is the focus of this paper), there will be three nonzero RHS terms remaining. This will require a more complicated procedure to arrive at an arbitrary RHS vector.

The original intent was to use this approach as a parallelized solver for banded systems. However, it was found that such solutions provide insight into the systems themselves and can lead to novel solutions. For example, the finite difference form of the Euler-Bernoulli equation leads to a series of evanescent solutions having compact support that can be appropriately superimposed to obtain vibration confinement for any arbitrary load vector. Specifically, we found that this process could lead to vibration confinement with only one load term at each boundary of the confined region. Conventional approaches, on the other hand, require two terms at adjacent nodes at each boundary.

A fluid-loaded beam (i.e., a beam in water) is more complex because of acoustic fields that impose loads on the entire beam. It is notable that past work has primarily focused on vibrating structures acting either *in-vacuo* or in-air. As a result, the fluid loading on the structure is often negligible and does not influence the mode shapes and structural dynamic response. Related work by Guiguo and Fuller [14] addresses the specific problem of active control of sound radiation from a semi-infinite beam with a clamped edge. Their approach is restricted to light fluid loading where the acoustic medium does not influence the beam structural dynamics. Other approaches for the light fluid loading case have been based on minimization of the local volume displacement into the fluid to control acoustic radiation [15]. In addition to the unique mathematical aspects of the present study, the control of sound radiation from a beam into a heavy fluid medium is noteworthy.

Because of the fluid loading, it is not possible to use the solution approach developed for banded linear systems. It is also not possible to completely achieve vibration confinement with just two actuators. However, we have been able to form a weighted sum of solutions with compact support that achieves near-confinement. We can either set the displacement amplitude outside of the confined region to zero, which would result in a small amplitude load distribution outside of the confined region, or we can set the load vector to zero outside the confined region, which would result in a continuous (but small amplitude) displacement distribution outside of the confined region.

This paper is organized as follows. We make use of a forward/backward solution approach in Section 2 to solve the beam vibration problem in air, obtaining an evanescent displacement vector and a load vector having compact support. We then develop a weighted sum of such solutions to achieve vibration confinement. We analyze the response to perturbations to that solution, and develop an analytical model that also demonstrates confinement with only two actuators on a uniform beam. In Section 3, we consider fluid-loaded beams. Here we generate solutions that emulate the evanescent displacement vector in air, and then consider other solutions having compact support, i.e., involving only one nonzero displacement term. We then perform a weighted sum of such solutions to generate near-confinement with only one actuator at either end of the confined region. Here the amplitude is small but nonzero outside the confined region as a consequence of the loads imposed by the acoustic field. Finally we present conclusions in Section 4.

2. Vibration confinement without fluid loading

2.1. Numerical algorithm

As mentioned in the introduction, our approach for non-fluid-loaded beams is based on a numerical algorithm [12,13] for banded linear systems that we apply to the beam vibration problem. We use this algorithm in lieu of a conventional solver because it automatically generates an evanescent solution (exhibiting an exponential growth/decay response) and a load vector having compact support. We will show that such solutions can support vibration confinement with only a single load term (representing an actuator) at either end of the confined region. In contrast, the use of a conventional solver would lead to *two* actuators at either end of the confined region for a pentadiagonal system [6].

We model the Euler-Bernoulli equation, i.e.,

$$EI\frac{\partial^4\Psi}{\partial x^4} + \rho A\frac{\partial^2\Psi}{\partial t^2} = W. \tag{1}$$