Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Randomly-disordered-periodic-induced chaos in a piezoelectric vibration energy harvester system with fractional-order physical properties

Di Liu [a,*], Yong Xu [b], Junlin Li [c]

[a] *School of Mathematical Sciences, Shanxi University, Taiyuan 030006, China*
[b] *Department of Applied Mathematics, Northwestern Polytechnical University, Xi'an 710072, China*
[c] *School of Applied Science, Taiyuan University of Science and Technology, Taiyuan 030024, China*

A R T I C L E I N F O

Article history:
Received 17 October 2016
Received in revised form
17 March 2017
Accepted 18 March 2017
Handling editor: L.G. Tham

Keywords:
Vibration energy harvesting
Fractional order deflection
Randomly disordered periodic
Random Melnikov method
Chaos

A B S T R A C T

The chaotic behavior of the piezoelectric vibration energy harvester (VEH) system with fractional order physical properties under randomly disordered periodic excitations is investigated. By using random Melnikov method, a mean square criterion is used to detect the necessary conditions for chaotic motion of this stochastic system. The results indicate that the increase of the noise intensity will result in the occurrence of chaos and the changes of the possible chaotic region in phase space, which first enlarging and then shrinking with a change in trend. The threshold of amplitude of randomly disordered periodic for the onset of chaos is determined by the numerical calculation via the largest Lyapunov exponent. The effects of noise intensity on chaos are also investigated through the largest Lyapunov exponent, phase portraits, Poincaré maps. At the same time, the effects of intensity of random frequency on the mean square voltage are further discussed, which show that the square voltage is in positive proportion to the size of the chaotic region. It is demonstrated that the essential changes of the dynamical behavior of the piezoelectric energy harvester system with fractional order physical properties will occur through changing the noise intensity, which can not only induce or suppress the onset of the chaos, but also raise or reduce the mean square voltage. Finally, the 0–1 test of responses is used to quantify the responses of VEH system, which further supports the effects of noise intensity on chaotic behavior of the system.

1. Introduction

Vibration energy harvesters (VEH) devices are designed to transform available ambient energy into usable low-consumed electronics through various transduction mechanisms, such as piezoelectricity and electromagnetic induction [1–3]. They are always designed for small electronic devices, to recharge the batteries and to solve battery replacement problematic. Hence, VEH devices have become an outstanding research field and have received the scholar's continual attention. One important goal now is to harvest maximizing the output power from environmental vibration by optimizing the physical properties of the material, the design of the device, and advantage of the random disturbance of the work environment.

* Corresponding author.
E-mail address: di-lau@hotmail.com (D. Liu).

http://dx.doi.org/10.1016/j.jsv.2017.03.018

In support of this goal, the potential benefits of nonlinearity restoring force of material have been introduced to increase available power in VEH device [3–5]. In fact, the Duffing oscillator, as the simplest nonlinearity model, was used in the design of many VEH devices, with the addition of electromechanical coupling for the harvesting circuit. For example, Mann and Owens [6] used the magnetic interactions to create an electromagnetic VEH system of nonlinear bistable potential well and to validate the potential well escape phenomenon can be used to broaden the frequency response by theory and experiments. Erturk and Inman [7] constructed a nonlinear piezomagnetoelastic VEH system, and investigated numerically and experimentally the responses of the system under harmonic excitation. Stanton et al. [8] used the method of harmonic balance to study the responses of the bistable piezoelectric inertial generator, and analyze the vibration stability of the system in the presence of varying harmonic environmental loading.

Since randomness exists in most real-world circumstances widely and may significantly change the dynamic behavior of VEH systems, the effects of random excitations have attracted a lot of researchers' attention. McInnes et al. [9] found that the stochastic resonance will significantly improve the bistable harvester performance compared to intrawell vibrations through numerical simulation. Adhikari et al. [10] analyzed the mean power of a linear single-degree-of-freedom piezoelectric VEH model subject to Gaussian white noise. Gammaitoni et al. [11] numerically and experimentally studied the responses of a monostable piezoelectric VEH system under Gaussian white noise excitation, and revealed that nonlinear oscillators can outperform the linear case in the context of random influence. Later, Litak et al. [12] calculated the stochastic responses of a nonlinear piezomagnetoelastic VEH oscillator by Gaussian white noise. Comparing with the numerical calculation method, some stochastic approximation methods have been developed and used to investigate the VEH device subject to random excitations. Green et al. [13] found that the Duffing-type nonlinearities can be used to reduce the size of stochastic electromagnetic VEH devices and without affecting its power output through the technique of equivalent linearization. Jiang and Chen [14] used the Van Kampen expansion method to derive the statistical moments of the response of the piezoelectric coupling system and discussed the effects of the system parameters on the response moments. Daqaq [15] introduced the moment method to investigate the voltage response statistics of VEH system, and discovered the time constant ratio of the VEH system plays a key role in developing the performance of nonlinear harvesters. Whereafter, He and Daqaq [16,17] illustrated the effects of the potential energy function on the mean steady-state approximate output power through the statistical linearization techniques. Recently, the different kinds of the stochastic averaging methods have been already developed to analyze nonlinear VEH system subjected to stochastic excitation [18–21].

Chaos is a kind of seemingly random, chance or irregular movement, which appears in nonlinear determined systems and widely exists in nonlinear VEH systems. The essence of chaos is sensitive to initial conditions for the long-term behaviors, which will result in unpredictability. Many effective approaches have been proposed to determine the occurrence of chaos of VEH systems. Stanton et al. [22] expanded the Melnikov theory recently to discuss the performance of a bistable VEH system in the presence of broadband excitation by analyzing the critical condition for homoclinic bifurcation. Harne et al. [23] derived the critical conditions for high-energy orbit of a harmonically excited bistable VEH system by the same method. When consider the inherent characteristics of piezoelectric materials, especially viscoelastic, Cao et al. [24,25] studied the chaotic dynamics of broadband piezoelectric VEH system with fractional damping and additional magnets. In addition, various fractional order power laws on stress-strain relation had been experimentally obtained for flexible materials [26–28], which are important materials of piezoelectric device and hence get a worldwide attention [29–31]. Recently, the chaotic responses and output powers of VEH system with fractional order power law on stress-strain relation had been researched by Kwuimy at el. [32]. So far, to the authors' knowledge, few scholars pay attention to stochastic VEN system with fractional order power law on stress-strain relation. To address the lacks of research in this aspect, the present work develops the random Melnikov method to analyze the effect of randomly disordered periodic on the chaos of a piezoelectric VEH system.

The paper is organized as follows. Section 2 considers the model of a piezoelectric VEH with fractional order physical properties under randomly disordered periodic excitation. Section 3 presents the threshold of randomly disordered periodic amplitude for the onset of chaos in the system using the random Melnikov method. Section 4 demonstrates the largest Lyapunov exponent, phase portraits, Poincaré maps and 0–1 test of responses of VEH, which are used to verify the analytical results. At the same time, the effects of intensity of random frequency on the mean square voltage are discussed. The last section is conclusion.

2. Mathematical model and randomly disordered periodic load

Base on the Euler-Bernoulli beam and Lagrange equation, we consider the transduction mechanism due to the piezoelectric properties of ceramic patches at the ends of the vertical flexible beam, as shown in Fig. 1, whose equation of motion can be expressed as

$$\begin{aligned} &M\ddot{X} + C\dot{X} + \frac{dU}{dX} - \Theta V = M_z\ddot{Z}, \\ &C_p\dot{V} + \frac{1}{R}V + \Theta\dot{X} = 0 \end{aligned} \tag{1}$$