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Nonlinear vibration of viscoelastic beams described using fractional order derivatives

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A B S T R A C T

The problem of non-linear, steady state vibration of beams, harmonically excited by harmonic forces is investigated in the paper. The viscoelastic material of the beams is described using the Zener rheological model with fractional derivatives. The constitutive equation, which contains derivatives of both stress and strain, significantly complicates the solution to the problem. The von Karman theory is applied to take into account geometric nonlinearities. Amplitude equations are obtained using the finite element method together with the harmonic balance method, and solved using the continuation method. The tangent matrix of the amplitude equations is determined in an explicit form. The stability of the steady-state solution is also examined. A parametric study is carried out to determine the influence of viscoelastic properties of the material on the beam's responses.

1. Introduction

Fractional calculus and fractional differential equations have more and more frequently been used in modern viscoelasticity problems. Literature surveys, together with bibliographical information, are now available in [1,2]. In particular, an overview of new trends and the results of applications of fractional calculus in dynamical problems are provided in [2]. The dynamic analysis of structures with viscoelastic layers or ones made of viscoelastic materials is the subject of numerous papers [1,3–19]. Viscoelastic materials are described using different rheological models, for instance, models with fractional derivatives [1,3,5,10,11,19] are used.

Linear dynamic analysis of fractionally damped beams and bars rods is the subject of paper [20], where the multiple scales method is used to develop the amplitude equations and determine the stability boundaries in the case of parametric excitations.

In paper [5], the dynamics of sandwich beams is analysed in the time domain and the fractional Zener model is used to describe the properties of the viscoelastic layer. The steady-state nonlinear vibrations of viscoelastic arches are investigated by Leung et al. [3]. The arches' material was modelled by the Kelvin-Voigt model with fractional derivatives and the region of main resonances is analyzed in particular. A single mode is used to generate nonlinear differential equations with fractional derivatives. The so-called residue harmonic balance method is used to obtain solutions of the first and higher orders to the above-mentioned equation. Multiple bifurcation solutions, the jump phenomenon and the saddle-node are observed. The nonlinear free and forced vibration of sandwich plates with the incompressible viscoelastic core is

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investigated in [7]. The standard linear model was used to describe the viscoelastic part of the plate. The method of multiple scales is applied to solve the equations of motion and to analyze the plate's behaviour in the primary resonance region. The rheological models with fractional derivatives were used in [10,11] to describe the viscoelastic dampers and viscoelastic layers. The dynamic characteristics of the frame structures or multilayered beams are investigated and the results obtained for dampers modelled in different ways are compared in [10]. The standard Biot model was used in [18] to analyze the viscoelastic structures.

In a series of papers [4,6,21,22], the nonlinear steady-state vibration of sandwich beams was also analyzed by Daya et al. In those papers, the harmonic balance method is used to obtain one-harmonic steady-state solutions to the problem under consideration. The nonlinear vibration of beams made of the viscoelastic Kelvin-Voigt material is also investigated in [23,24]. The Prony–Dirichlet series and the multiple scales method are used in [13] to study both the linear and nonlinear vibration of composite plates. Asymptotic methods are often used in the dynamic analysis of beams and plates. The current description of these methods and numerous examples are reported in monograph [25].

The nonlinear vibration of fractionally damped viscoelastic composite beams is considered in [26] where a single mode solution is obtained analytically using the multiple-time scales method. A constitutive equation, similar to the one used in the Kelvin model, is adopted. Both the free and steady-state vibrations are considered. The dynamical analysis of nonlinear viscoelastic plates under subsonic flow and external loads are considered in [27]. The multiple scales method is used to solve the equation of motion. The response curves for primary as well as super- and subharmonic resonances are determined.

The linear equation of motion of the Euler-Bernoulli beam made of a viscoelastic material, described using the fractional derivatives, was analyzed in detail by Di Paola et al. [28]. It was assumed that the stress-strain relationship contains the Caputo's fractional derivative whereas the inverted relationship contains the Riemann–Liouville fractional integral. The parametric vibration of an axially moving beam made of a fractional-order Kelvin material is considered in [29]. In particular, the parametric resonance together with stability of motion was investigated.

The steady-state vibration of a one-degree-of-freedom system, described by the Duffing nonlinear equation with fractional derivatives, was investigated in [30–33]. The averaging method or the harmonic balance method was used to determine the solution.

In the present paper, the problem of nonlinear, steady-state vibration of viscoelastic beams is considered. It is assumed that the beams' viscoelastic material can accurately be modelled using the Zener model. The constitutive equation of the model under consideration contains fractional derivatives. As stated in [34], the Zener model is the simplest one and it preserves all the main properties of real viscoelastic materials. However, the constitutive equation of the model contains derivatives of both stress and strain. This produces some additional difficulties in the context of nonlinear dynamic problems, in comparison with the Kelvin model (used in [3]) or with the model suggested by Di Paola et al. [28]. Beams with immovable ends are considered and the von Karman theory is used to describe the effects of geometrical nonlinearity. The steady state vibration of the beams are described using the one-harmonic function of time. The harmonic balance method together with the finite element method is used to derive the amplitude equations which are solved by the continuation method. Moreover, the results of representative calculations are described and briefly discussed.

The paper consists of eight sections. Section 2 summarizes the main equations which describe the behaviour of the beam undergoing large amplitude vibration, whereas Section 3 describes the assumed steady-state solution. The discrete form of the amplitude equations is derived in Section 4 and the continuation procedure used to determine the response curves is briefly presented in Section 5. Stability of the steady-state solution is examined in Section 6. The results of calculation are discussed in Section 7. The concluding remarks are presented in Section 8. Some useful formulae are given in Appendix A.

2. Description of the beam

According to the Euler-Bernoulli theory of beams, the horizontal displacement $u_x(x, z, t)$ and the transversal displacement $u_z(x, z, t)$ of a freely chosen point of the beam, of which the coordinates are (x, z), can be written in terms of the horizontal $u(x, t)$ and transversal $w(x, t)$ displacements of a neutral axis as follows:

$$u_x(x, z, t) = u(x, t) - zw_{,x}(x, t), \quad u_z(x, z, t) \equiv w(x, t). \tag{1}$$

where $w_{,x}(x, t) = \partial w/\partial x$ and t is time.

The generalized strain field is described as follows:

$$\varepsilon_x(x, z, t) = \varepsilon(x, t) - z\kappa(x, t), \quad \kappa(x, t) = -w_{,xx}(x, t), \tag{2}$$

where $\varepsilon_x(x, z, t)$ is the strain at an arbitrary point of a cross-section, $\varepsilon(x, t)$ is the linear strain of the neutral beam axis and $\kappa(x, t)$ is the beam's curvature.

According to the von Karman theory, the linear strain of neutral fibre is given by

$$\varepsilon(x, t) = u_{,x}(x, t) + \frac{1}{2}w_{,x}^2(x, t). \tag{3}$$

Well known equations of motion of the beam, resulting from equilibrium conditions, are: