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Wave propagation in axially moving periodic strings

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ABSTRACT

The paper deals with analytically studying transverse waves propagation in an axially moving string with periodically modulated cross section. The structure effectively models various relevant technological systems, e.g. belts, thread lines, band saws, etc., and, in particular, roller chain drives for diesel engines by capturing both their spatial periodicity and axial motion. The Method of Varying Amplitudes is employed in the analysis. It is shown that the compound wave traveling in the axially moving periodic string comprises many components with different frequencies and wavenumbers. This is in contrast to non-moving periodic structures, for which all components of the corresponding compound wave feature the same frequency. Due to this "multi-frequency" character of the wave motion, the conventional notion of frequency band-gaps appears to be not applicable for the moving periodic strings. Thus, for such structures, by frequency band-gaps it is proposed to understand frequency ranges in which the primary component of the compound wave attenuates. Such frequency band-gaps can be present for a moving periodic string, but only if its axial velocity is lower than the transverse wave speed, and, the higher the axial velocity, the narrower the frequency band-gaps. The revealed effects could be of potential importance for applications, e.g. they indicate that due to spatial inhomogeneity, oscillations of axially moving periodic chains always involve a multitude of frequencies.

1. Introduction

The present paper addresses the analysis of transverse waves propagation in an axially moving string with a periodic variation in elastic and inertial properties. Due to their high technological importance, similar problems concerned with the dynamics of axially moving materials have received considerable attention from many researchers, see e.g. [1–10]. Transmission chains, drive belts, thread lines, paper machines, band saws, and fibers are some of the technological applications. Most of the existing studies have addressed the constant axial transport velocity problem, however, dynamics of elastic structures moving with a time-dependent axial velocity has been also considered, e.g. by Miranker [2] and more recently in the papers [5,7,8].

Real technological axially moving systems as the above-mentioned are inherently *inhomogeneous*, i.e. feature spatial modulations of their parameters. However, effects of such modulations on the systems response are not yet fully uncovered though of potential importance for applications. The present paper addresses the dynamics of a *non-uniform* string moving

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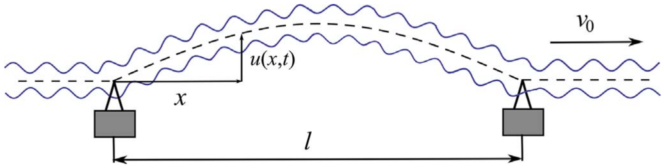

Fig. 1. The considered non-uniform string moving axially with a given constant velocity v_0.

axially with a constant velocity. The string is assumed to be periodic in the spatial coordinate, as is relevant to describe real systems, so that a structure involving *coupled spatial-temporal modulations* is considered. Studies of the dynamics of structures with space and time varying parameters have gained much attention in the recent years (e.g., [11–17]), and coupled spatial-temporal modulations were shown to be a potential tool to tailor effective dynamic properties of structures. The problem under consideration is of particular importance for applications involving diesel engines since the string effectively models roller chain drives for such engines by capturing both their spatial periodicity and axial motion.

A novel analytical approach, the *Method of Varying Amplitudes* (MVA) [18,19], is employed in the paper. This approach may be considered a natural continuation of the classical methods of harmonic balance [20] and averaging [21–23]. It implies representing a solution in the form of a harmonic series with varying amplitudes; however, in contrast to the asymptotic methods, the amplitudes are not required to vary slowly. It is strongly related also to Hill's method of infinite determinants [20,24,25], and to the method of space-harmonics [26].

2. Governing equations

The kinetic energy of a non-uniform string with length l moving axially with a given constant velocity v_0 is:

$$T=\int_0^l \frac{1}{2}\rho A(x,t)\left[\left(\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial x}\right)^2+v_0^2\right]dx, \tag{1}$$

where ρ is the density of the string material, $A(x,t)$ the varying cross-sectional area, and $u=u(x,t)$ the transverse displacement of the string's cross section at time t located at distance x from the left boundary, see Fig. 1. Note that due to the axial motion of the string, its cross-section at a given x changes with time, so that $A(x,t)$ depends both on time t and spatial coordinate x. Motion of the string is considered with respect to the not moving coordinate system that dictates the form of the time derivative in (1).

Assuming the slopes of transverse deflections of the string to be small, and neglecting higher order terms, we obtain the following expression for the potential energy of the string [2,3]:

$$V=\int_0^l \frac{P_0}{2}\left(\frac{\partial u}{\partial x}\right)^2 dx, \tag{2}$$

where P_0 is the tension of the string, assumed to be constant. According to Hamilton's principle we have:

$$\delta H=\delta\int_{t_1}^{t_2} L dt=0, \tag{3}$$

where $L=T-V$ is the Lagrangian of the system. The boundary conditions for transverse motions of the string are [2]:

$$u(0,t)=u(l,t)=0. \tag{4}$$

Eq. (3) can be rewritten in the form:

$$\delta H=\int_{t_1}^{t_2}\int_0^l \delta h dx dt=\int_{t_1}^{t_2}\int_0^l\left(\frac{\partial h}{\partial \dot u}\delta\dot u+\frac{\partial h}{\partial u'}\delta u'\right)dx dt=0, \tag{5}$$

where dots and primes denote derivatives with respect to time t and spatial coordinate x, respectively, and

$$h=\frac{1}{2}\rho A(x,t)\left[\left(\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial x}\right)^2+v_0^2\right]-\frac{1}{2}P_0\left(\frac{\partial u}{\partial x}\right)^2. \tag{6}$$

Taking into account that $[\delta u]_0^l=0$ and $[\delta u]_{t_1}^{t_2}=0$, (5) gives:

$$\delta H=-\int_0^l\int_{t_1}^{t_2}\left[\rho\left(\frac{\partial A}{\partial t}+v_0\frac{\partial A}{\partial x}\right)\left(\frac{\partial u}{\partial t}+v_0\frac{\partial u}{\partial x}\right)+\rho A\left(\frac{\partial^2 u}{\partial t^2}+2v_0\frac{\partial^2 u}{\partial x\partial t}+v_0^2\frac{\partial^2 u}{\partial x^2}\right)-P_0\frac{\partial^2 u}{\partial x^2}\right]\delta u dx dt=0 \tag{7}$$

So that the following equation describing transverse oscillations of the non-uniform axially moving string is obtained: