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A mixed-contact formulation for a dynamics simulation of flexible systems: An integration with model-reduction techniques

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ABSTRACT

A new numerical procedure for efficient dynamics simulations of linear-elastic systems with unilateral contacts is proposed. The method is based on the event-driven integration of a contact problem with a combination of single- and set-valued force laws together with classical model-reduction techniques. According to the contact state, the developed event-driven integration enables the formulation of reduced system matrices. Moreover, to enable the transition among different reduced spaces the formulation of the initial conditions is also presented. The method has been developed separately for each of the four most popular model-reduction techniques (Craig–Bampton, MacNeal, Rubin and dual Craig–Bampton). The applicability of the newly presented method is demonstrated on a simple clamped-beam structure with a unilateral contact, which is excited with a harmonic force at the free end.

1. Introduction

Numerical investigations of systems represent a common engineering technique to reduce the costs of an end-product. The dynamic analysis of systems is usually conducted by applying a dense mesh and using the classical finite-element method. As the refinement of the finite elements increases, a large computational effort is required to solve systems with a large number of degrees of freedom (DoF). This is especially the case when a transient time response is required, due to the usually high stiffness of mechanical systems, which implies small time-steps during the integration. Another well-known problem is the modelling of contacts between flexible bodies [1–5]. These contacts introduce non-linearities and add a high stiffness in the contact region, which further decreases the required time-step. This often leads to long computation times that require enormous computational resources.

The model-reduction techniques [6] are well-known methods that address the large number of DoF. They reduce the system matrices, but retain the essential information for the analyses. They are mostly used for analysing small deformations and vibration phenomena. In the case of the system response, they also make possible significantly faster integration times, due to the reduced number of equations of motion (EoM). The most common methods are the Craig–Bampton method [7], MacNeal [8], Rubin [9], Craig–Chang [10] and the dual Craig–Bampton method [11]. A practical case and a

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comparison of the methods can be found in [12]. In recent years, in addition to the classical linear model-reduction techniques, the non-linear model-reduction techniques have also attracted a lot of scientific attention [13]. They enable the modelling of non-linearities, such as large deformations, local non-linearities, non-linear damping or coupling effects, for instance the analysis of jointed structures [14].

The modelling of contacts between flexible bodies is usually formulated using the penalty method [5] or more advanced methods that introduce a non-penetration condition [15,16]. By using the penalty method, a large penalty factor may lead to an ill-conditioned stiffness matrix and consequently to a poor convergence. On the other hand, the method that imposes a non-penetration condition originates from the contact formulation between rigid bodies [5,17]. The first studies were made on rigid-body unilateral contacts in the form of the linear complementarity problem (LCP) and were published by Lötstedt [18]. In the framework of non-smooth contact dynamics, Moreau [19] introduced a numerical treatment of rigid bodies with unilateral contacts, Coulomb friction and impacts. Without any significant change to the computational strategies Jean [20] applied a method to treat the contacts between flexible bodies. In comparison to time-stepping methods, the event-driven methods integrate the dynamical system until the event occurs. In [21], Čepon and Boltežar proposed a mixed-contact formulation between flexible bodies using event-driven integration together with the penalty method. The method models the impact with the penalty method, while the continuous contact is modelled by a Lagrange-multipliers method. In the recent years newer methods have emerged, which present a more advanced (non-linear) contact model. Willner [22] and Goerke and Willner [23] present an iterative elastic half-space solution based on a variational principle. A simple modification also enables the approximative solution of the elasto-plastic contact, which is modelled with a power-law relationship between the pressure and the contact gap. Arz and Laville [24] presented an impact model that is based on a system of two equations: Newton's second law and the non-linear viscoelastic contact force law, originally presented in [25]. Pohrt and Popov [26] observed a non-linear behaviour of the contact normal stiffness, which is in a power-law dependence with the normal force. Furthermore, they also observed power-like dependencies on the rms value of the roughness, the elastic modulus and the nominal area of contact. The power-law dependence was confirmed experimentally by Zhai et al [27]. Kostek [28] presented a non-linear contact model describing the hysteresis of a dry contact for rough surfaces loaded in the normal direction. The model also accounts for the plastic deformation of the virgin contact as well as the insensibility of the contact hysteresis to the frequency of the loading. Ahmadian and Mohammadali [29] consider the hysteresis effects in both normal and shear contact directions. The model is rate independent and represents coupling effects between normal and shear displacements and is based on power-law dependence between the normal/shear force and the displacement.

In this paper a new method is proposed that enables efficient dynamics simulations of linear-elastic systems with unilateral contacts. The method proposes the integration of a contact formulation, as presented in [21] together with classical modal-reduction techniques. According to the contact state, the developed event-driven integration scheme enables the formulation of reduced system matrices. The updating algorithm for the static and vibration modes is therefore in direct correlation with the changing boundary DoF during the system response. It is shown that the reduction basis for the continuous and the impact contact formulation can be obtained based on a flexible system without contacts. Moreover, to enable the transition among different reduced spaces the formulation of the initial conditions is also presented. The method has been developed separately for each of the four most popular model-reduction techniques (Craig–Bampton, MacNeal, Rubin and dual Craig–Bampton). The applicability of the newly presented method is demonstrated on a simple clamped-beam structure with a unilateral contact, which is excited by a harmonic force at the free end.

The paper is organised as follows. The second section presents the four classical model-reduction techniques. The third section presents the event-driven integration of reduced models with unilateral contacts. The new definitions of the system matrices, the initial conditions and the events are defined according to the contact state. The fourth section presents a case study of a clamped beam with a unilateral contact, together with the advantages of the proposed method. In the last section a summary and the contributions are presented.

2. Model reduction techniques

Model-reduction techniques [30] are efficient methods to reduce the size of large finite-element method (FEM) models. They retain the dense finite-element mesh, but replace the physical degrees of freedom with a much smaller set of generalised degrees of freedom. This is done by modal superposition and truncation. The methods can be divided into two main groups [31]: fixed- and free-interface methods. The most known fixed-interface method is the Craig–Bampton method [7], and the free-interface methods are MacNeal [8], Rubin [9] and dual Craig–Bampton [11]. A good overall step-by-step description of the methods can be found in [12]. The methods consist of a reduction basis containing static modes and a limited number of vibration modes. The static modes can be further divided into the constraint, attachment and residual attachment modes. The vibration modes are divided into the free-interface, rigid-body and fixed interface modes. Hence, the fixed/free-interface methods are determined by the selection of the vibration modes and the accompanying static modes. A detailed description of the above-mentioned modes is found in [12,32,31].

The model-reduction techniques are closely connected to the substructuring field, where a substructure dynamical model is defined as:

$$\mathbf{M}^{(s)}\,\ddot{\mathbf{u}}^{(s)}(t)+\mathbf{C}^{(s)}\,\dot{\mathbf{u}}^{(s)}(t)+\mathbf{K}^{(s)}\,\mathbf{u}^{(s)}(t)=\mathbf{f}^{(s)}(t)+\mathbf{g}^{(s)}(t), \tag{1}$$

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