Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Pressure-based integral formulations of Lighthill–Curle's analogy for internal aeroacoustics at low Mach numbers

N. Papaxanthos [a,*], E. Perrey-Debain [a], S. Bennouna [a,b], B. Ouedraogo [a],
S. Moreau [a], J.M. Ville [a]

[a] *Laboratoire Roberval UMR CNRS 7337, Sorbonne Universités, Université de Technologie de Compiègne, CS 60319, 60203 Compiègne, France*
[b] *Valeo Thermal Systems, 8 rue Louis Lormand, 78320 La Verrière, France*

ARTICLE INFO

Article history:
Received 18 July 2016
Received in revised form
7 January 2017
Accepted 17 January 2017
Handling Editor: A.V. Metrikine

Keywords:
Aeroacoustics
Low Mach
Internal flows
Lighthill's analogy
BEM
Large eddy simulation
HVAC noise

ABSTRACT

The use of unsteady incompressible-flow simulations has become very popular for aeroacoustic noise predictions at low Mach numbers, as it provides a good compromise between computational time and reliable predictions. The acoustic radiation of the aerodynamic sources is calculated in a second step by solving an appropriate system of acoustic equations. In order to predict the noise produced by confined flows, two integral formulations of Lighthill–Curle's analogy are developed. Both formulations require only the knowledge of the incompressible-flow pressure. The first one, which is based on Ribner's reformulation of Lighthill's source terms, is exact and shall serve as a reference to the second approximate formulation which involves only the pressure on the boundary of the fluid domain. The two formulations are shown to be in excellent agreement for the case of a long straight duct obstructed by a diaphragm which makes the simplified integral formulation a reliable alternative to usual computational methods. The sound power levels as well as the modal contributions compare favorably with measurements. Moreover, it is shown that the computed radiated sound is independent of the outlet condition of the flow simulation.

1. Introduction

The prediction from the conception phase of the generated sound by a turbulent flow inside ducts, such as heating, ventilation and air-conditioning (HVAC) systems or pipelines, is a major competitive challenge for designers. In the car industry, the development of silent motorization as for hybrid or electric vehicles has made the noise resulting from air conditioning systems a dominant one in the vehicle. In this respect, our concern deals with the presence of confined obstacles with low Mach number flows that results in unsteady aerodynamic fluctuations which generate sound waves propagating along the ducting.

Computational aeroacoustics (CAA) has achieved substantial progress over the past decades especially through advances in computational fluid dynamics (CFD) and the development of processor performance. CAA becomes in industries more used than semi-empirical predictions based on power laws established by Lighthill [1] or by others later [2,3]. Using purely

---

[*] Corresponding author.
E-mail address: nicolas.papaxanthos@utc.fr (N. Papaxanthos).

http://dx.doi.org/10.1016/j.jsv.2017.01.030

Please cite this article as: N. Papaxanthos, et al., Pressure-based integral formulations of Lighthill–Curle's analogy for internal aeroacoustics at low Mach numbers, *Journal of Sound and Vibration* (2017), http://dx.doi.org/10.1016/j.jsv.2017.01.030

computational means, a hierarchy of methods, ranging from stochastic noise generation (SNGR) with unsteady Reynolds-averaged Navier–Stokes (RANS) based turbulence models at the lower end [4] to the resolution of all flow scales with the direct numerical simulation (DNS) at the higher end [5], provide estimation of the radiated sound from unsteady-type flows. At low Mach and high Reynolds numbers, the unsteady incompressible-flow simulation coupled with an acoustic analogy has proven to be a good compromise with reasonable computational time and accurate prediction [6]. This two-step resolution, called hybrid approach, exhibits reliable results for various industrial applications as the prediction of the noise emitted by simplified elements of HVAC systems [7,8], by trailing edges [9] or by suction valves [10]. A fundamental assumption for the hybrid approach-based prediction is the one-way coupling of flow and sound, thus, the feedback mechanisms between the acoustic and aerodynamic fields are ignored in the analysis.

Modern aeroacoustics began in the early 1950s with Lighthill's analogy which brought to light equivalent acoustic quadrupole sources in the flow field as responsible for the flow noise [1]. Curle extended next the analogy to account for the presence of solid surfaces immersed in the unsteady flow field [11]. For low Mach number flows, Curle demonstrated that the scattered field by the rigid body dominates the direct field from quadrupoles if the surface is compact (i.e. of small dimensions compared to the acoustic wavelength). This finding has been largely used to calculate at little expense the radiated sound from flow/compact obstacle interaction [12,13]. Numerous studies have clarified this conversion of the incident field from quadrupoles into a dominant scattered field [14,15]. It is shown that the scattering effect is not restricted to compact obstacles and can also occur in the presence of non-compact surfaces with singularities, such as edges, protuberances or corners. Based on this observation, a recent simplification of the integral form of Lighthill's equation has been proposed [16,17]. Note that contrary to compact obstacles, non-compact surfaces also modify the sound radiation by reflexion and diffraction without any contribution to its generation.

The calculation of the flow noise with non-compact surfaces from incompressible CFD data can be accomplished in various manners. Lighthill's equation can be implemented in a finite element framework [18,19]. Green's function tailored to the geometry may be used to account for all the reflections and scattering, this latter can be found analytically for simple geometries [9,20] or numerically for more complex geometries [10,21]. Schram proposes a boundary element method (BEM) approach that computes the acoustic component of the wall pressure [22]. Martínez-Lera et al. solve Lighthill's equation through a boundary value problem for the scattered pressure [16]. In this paper, two integral formulations based on the mere knowledge of the incompressible-flow pressure are derived. In the first one, the classical volume term of Lighthill–Curle's analogy takes a more suitable form from a numerical point of view [23,24]. The second is based on a simplification which uses the fact that scattering phenomena account for most of the radiated sound power [16,17].

The derivation of the two formulations is presented in Section 2. Their validation is supported by a realistic test case: the aerodynamic noise due to the insertion of a diaphragm in a rectangular duct. Before the description of the flow simulation in Section 4, a short presentation of the experimental setup and procedure is made in Section 3. Section 5.1 shows the numerical results and the comparisons with the measurement of the sound power levels and the modal contributions. In Section 5.2, a spatial Fourier transform is performed on one surface of the duct and brings to light spurious pressure fluctuations from the CFD simulation depending on the outlet boundary condition. The approach which is developed in this paper is shown to be not sensitive to this numerical phenomenon and allows to produce consistent results regardless of the outlet condition in the flow modeling.

2. Integral formulations of Lighthill's equation

Lighthill's equation is an inhomogeneous Helmholtz equation satisfied by the pressure which is derived simply by rearranging the Navier–Stokes equations [1]. In the frequency domain (the Fourier convention $e^{i\omega t}$ is adopted), Lighthill's equation is:

$$\left(\Delta + k^2\right)p = q \quad \text{where } q = -\frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \tag{1}$$

is the source term and is composed of the double spatial derivative of Lighthill's tensor T_{ij}, $k = \omega/c_0$ is the wavenumber with c_0 being the sound velocity. For isentropic low Mach number flows, it is commonly accepted that Lighthill's tensor takes the simplified form $T_{ij} \simeq \rho_0 u_i u_j - \tau_{ij}$ with ρ_0 being the fluid density and u_i the i-component of the velocity. The viscous contribution τ_{ij} is usually neglected for high Reynolds number flows but it is left here as it is provided by an incompressible-flow modeling.

In Fig. 1 is sketched an arbitrary confined domain Ω bounded by two ducts from where the fluid enters and exits the domain. The interaction of the flow and obstacles or constrictions generates sound which propagates through the inlet and outlet ducts. In each duct, which are considered identical for simplicity, the pressure consists of acoustic waves only so it can be expressed as a sum of radiating modes:

$$p(\boldsymbol{x}) = \sum_m a_m \Phi_m(\boldsymbol{x}) \quad \text{on } \partial\Omega_\alpha \tag{2}$$

where the duct mode $\Phi_m(\boldsymbol{x})$ obeys the Helmholtz equation and a_m is a modal coefficient, α takes the value of 1 or 2 depending on the fictitious surface. Using the orthonormality of the modes and the fact that $\partial_n \Phi_m = -ik_m \Phi_m$ (k_m is the axial