Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Analytical investigation of machining chatter by considering the nonlinearity of process damping

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A R T I C L E I N F O

Article history:
Received 21 September 2016
Received in revised form
27 December 2016
Accepted 6 January 2017

Keywords:
Nonlinear chatter
Process damping

A B S T R A C T

In this paper, the well-established problem of self-excited vibrations in machining is revisited to include the nonlinearity of process damping at the tool and workpiece interface. Machining dynamics is modeled using a time-delayed system with nonlinear damping, and the method of averaging is used to obtain the amplitude of the resulting limit cycles. As a result, an analytical relationship is presented to establish the stability charts corresponding with arbitrary limit cycles in machining systems. The presented analytical solutions are verified using experiments and numerical solutions.

1. Introduction

Regenerative chatter is the self-excited vibration that develops during machining processes [1–3]. Prediction and avoidance of the instability of chatter is critical, because unstable vibrations will damage the tool and workpiece or will result in a poor surface finish. Tobias [2] and Tlusty [3] found that self-excited vibrations could develop in metal cutting processes as a result of the feedback between the vibrations in subsequent cuts. To study chatter stability in various machining operations, the process is usually modeled as a linear or nonlinear time-delayed system [4–11]. In linear chatter theories the structure of the tool and workpiece are modeled using linear dynamic models and the cutting forces are assumed to linearly depend on chip thickness. According to linear chatter theories, if the width of the chip that is being removed exceeds the stability threshold of the system, the amplitude of the vibration will increase unboundedly. In practice, however, unstable vibrations stabilize at a “finite amplitude” due to the inherent nonlinearity of cutting forces.

Hanna and Tobias [12] studied the effect of the nonlinearities that arise from the tool's structure as well as the cutting forces in turning processes. They assumed cubic nonlinearities in the stiffness of the tool's structure and a cubic time-delay term in the cutting force model to show that the typical characteristics of nonlinear systems such as jump phenomena could be observed in machining systems. Later, Tobias and Shi [13] showed that the nonlinearities of the structure can be relatively neglected compared to the strong nonlinearities of cutting forces. Tlusty and Ismail [14] developed a time-domain simulation for turning operations in order to study the nonlinearity of the cutting forces that are due to the periodic disengagement of the tool from the workpiece in well-developed chatter. In their work, a piecewise-linear model with multiple delay terms was used to model cutting forces. Grabec [15] used a friction model to describe the forces at the tool and workpiece interface and concluded that the strong nonlinearities of cutting forces can result in chaotic dynamics in machining. Lin and Weng [16] developed a nonlinear force model by incorporating the effect of large vibrations in the mechanics of chip formation. Nayfeh et al. [17,18] included cubic and quadratic stiffness and time-delay terms in the equation

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http://dx.doi.org/10.1016/j.jsv.2017.01.006

Please cite this article as: K. Ahmadi, Analytical investigation of machining chatter by considering the nonlinearity of process damping, *Journal of Sound and Vibration* (2017), http://dx.doi.org/10.1016/j.jsv.2017.01.006

Nomenclature

F_y	Cutting force in the feed direction [N]
K_{fc}	Cutting force coefficient in the feed direction [N/mm^2]
h	Uncut chip thickness [mm]
b	Uncut chip width [mm]
K_{fe}	Edge force coefficient in the feed direction [N/mm]
ω_n	Modal angular velocity [rad/sec]
Y_0	Static contraction of the spring [mm]
F_s	Static cutting force [N]
m	Modal mass [Kg]
s_t	Static chip thickness, which equals to feedrate [mm/rev]
k	Modal stiffness [N/mm]
$y(t)$	General coordinate of the SDOF system [mm]
t	Time [sec]
T	Spindle revolution period [sec]
F_d	Dynamic portion of the cutting force [N]
ξ	Damping ratio of the SDOF system
F_p	Ploughing force [N]
K_{sp}	Specific indentation force coefficient [N/mm^3]
V	Volume of the extruded material [mm^3]
S	The cross-section area of the extruded volume [mm^2]
l_w	Length of flank wear [mm]
β	Clearance angle [rad]
V_c	Cutting speed [mm/sec]
D	Workpiece diameter [mm]
c_p	Process damping coefficient [N]
$\bar{y}$	Non-dimensional vibration coordinate
w	Non-dimensional width of cut
$\bar{c}$	Non-dimensional process damping coefficient
$\bar{T}$	Non-dimensional spindle revolution period
λ	Characteristic exponent
ω	Angular velocity of oscillations [rad/sec]
i	Imaginary unit
c_{eq}	Equivalent linear viscous damping coefficient [N.sec/mm]
η_{max}	Maximum slope of undulation [rad]
w_{min}	Non-dimensional width of cut at the minimum point of stability charts
ω_{min}	Oscillation frequency at the minimum point of stability charts [rad/sec]
SDOF	Single Degree Of Freedom
DDE	Delay Differential Equation
AC	Alternating Component

of turning dynamics and used multiple scales and harmonic balance methods to generate the bifurcation diagrams of the system. The mentioned literature mainly concentrates on modeling the nonlinearities of cutting forces, but in the recent literature, limit cycles have also been reported due to the nonlinearity of the damping at the tool and workpiece interface [19–22]. The undulations on the machined surface become extruded under the cutting edge, generating an additional damping source known as process damping [23–25]. Although process damping is shown to be nonlinear in nature [26–28], it has been treated as linear damping in analytical chatter models [19,20,29]. Linear process damping models are unable to explain the limit cycles that are observed in low-speed cutting, in which process damping dominates the dynamics of the system.

In this paper, a SDOF system is used to model self-excited vibrations in turning by considering a nonlinear process damping model. The method of averaging is used to obtain the amplitude of the resulting limit cycle in the steady state. As a result, analytical relationships are presented to establish the stability charts of the system corresponding with arbitrary limit cycle amplitudes.

In the next section, the basic theory of regenerative chatter is described by considering a nonlinear process damping model. The linear and nonlinear stability analysis is presented in Section 3, followed by numerical and experimental validations of the solution in Section 4.

2. Machining dynamics

2.1. Regenerative chatter

A Single Degree Of Freedom (SDOF) model of the regenerative vibrations of machine tools is shown in Fig. 1. The tool and the workpiece move towards one another in the feed direction (Y) while the cylindrical workpiece rotates around its axis at the angular velocity of $\omega_s = 2\pi/T$. Chip is generated by the rigid body motion of the tool and the workpiece in the feed direction at the feedrate of s_t millimeters per revolution of the workpiece. To simplify the modeling and to concentrate on the nonlinearities caused by process damping, the tool is assumed to be fully rigid and the workpiece is assumed to only vibrate in the feed (Y) direction. In this derivation the workpiece was chosen to be the flexible component of the system to be consistent with the experimental setup that is used in Section 4; however, similar derivations also apply when the tool is flexible in the feed direction. To model the vibrations using a SDOF system, only one vibration mode is assumed to dominate the dynamics in the Y-direction.

In linear mechanistic cutting force models [1], the projection of the overall cutting forces in the feed direction, F_y, is assumed to be proportional to the width and thickness of the uncut chip—b and h, respectively:

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