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Magneto-elastic oscillator: Modeling and analysis with nonlinear magnetic interaction

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ABSTRACT

The magneto-elastically buckled beam is a classic example of a nonlinear oscillator that exhibits chaotic motions. This system serves as a model to analyze the motion of elastic structures in magnetic fields. The system follows a sixth order magneto-elastic potential and may have up to five static equilibrium positions. However, often the non-dimensional Duffing equation is used to approximate the system, with the coefficients being derived from experiments. In few other instances, numerical methods are used to evaluate the magnetic field values. These field values are then used to approximate the nonlinear magnetic restoring force. In this manuscript, we derive analytical closed form expressions for the magneto-elastic potential and the nonlinear restoring forces in the system. Such an analytical formulation would facilitate tracing the effect of change in a parameter, such as the magnet dimension, on the dynamics of the system. The model is derived assuming a single mode approximation, taking into account the effect of linear elastic and nonlinear magnetic forces. The developed model is then numerically simulated to show that it is accurate in capturing the system dynamics and bifurcation of equilibrium positions. The model is validated through experiments based on forced vibrations of the magneto-elastic oscillator. To gather further insights about the magneto-elastic oscillator, a parametric study has been conducted based on the field strength of the magnets and the distance between the magnets and the results are reported.

1. Introduction

The phenomenon of chaos has attracted a lot of attention since its proposition by Lorenz in 1963 [1]. There has been a huge development in the theory and application of chaos since then. In this manuscript, we revisit a classical problem that serves as the first experimental evidence for the existence of chaotic motions in structural mechanics. The accolade is accredited to the magneto-elastically buckled beam, proposed by Moon and Holmes in 1979 [2]. It has become a central example in chaos theory since then. Recently, the magneto-elastically buckled beam has been revisited by many authors [3–8], specifically for nonlinear energy harvesting applications.

The magneto-elastic oscillator consists of a ferromagnetic cantilever beam buckled between two external magnets and undergoing forced oscillations. The interaction between the magnets and the ferromagnetic beam is nonlinear and depends on the distance between the magnets and the field strength of the magnets. A tip magnet can be added to the beam to

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enhance the effect of magnetic interaction. The article [2] by Moon and Holmes discusses the case without tip magnet and it is assumed that the magnetic field is concentrated at the tip of the ferromagnetic beam. However, in articles related to energy harvesting, a tip magnet is added to enhance the nonlinear magnetic interactions.

In [2], apart from experiments, Moon and Holmes have also outlined the formulation of a mathematical model for the magneto-elastically buckled beam using Euler–Lagrange equations. The formulation starts with a continuum model, incorporating the effect of linear elastic and nonlinear magnetic forces. The partial differential equation corresponding to the continuum model is then converted to an ordinary differential equation through a single mode approximation. This approximated system has a sixth order magneto-elastic potential and can have either one, three or five static equilibrium positions [2]. But in most of the analysis carried out henceforth in [2], the sixth order term is often neglected and terms up to the fourth order are retained. Under such simplifications, the system follows the well-known Duffing equation and may have one or three equilibrium positions depending on the parameters.

Unlike the softening and hardening configurations, which are quite well-explored at that time, the Duffing equation, taken up by Moon and Holmes to model this system, describes an oscillator with two stable states – a bistable oscillator. This is achieved by taking a negative linear stiffness and a positive third order stiffness [9]. Under certain conditions of harmonic excitations, the response of the bistable system can oscillate between the two stable states in a non-resonant and chaotic manner [2,10]. The bistable configuration of the oscillator is widely explored both experimentally and numerically, especially in the context of energy harvesting [3–8].

Moon and Holmes have also presented some preliminary experimental evidence on the configuration having five equilibrium positions [2]. This configuration has three stable and two unstable static equilibrium positions and is known as the tristable configuration. Zhou et al. have demonstrated the superior performance of tristable configuration over the bistable configuration in energy harvesting recently in 2014 [11]. The tristable configuration has been gaining momentum since then and has been the subject of various studies related to energy harvesting [12,13].

Experiments and numerical models based on data fit from experiments are abound in the literature for both the bistable and tristable configurations of the oscillator. These models generally utilize the non-dimensional form of equations corresponding to a forced oscillator with nonlinear restoring forces. The non-dimensional constants to be used in the equation are derived either from experiments [3,4,7,14] or from numerical simulations [14]. Under such cases, it becomes difficult to trace the effect of change in a physical parameter, say, the distance between the magnets, on the behavior of the system. An analytical model, relating the physical parameters to the system dynamics, would be helpful in such parametric studies. But, such a complete mathematical model with closed form expressions for the nonlinear restoring forces is yet to be realized.

In this manuscript, we seek closed form expressions for the nonlinear restoring forces and develop a mathematical model for the magneto-elastic oscillator to get further insight about its behavior. We try to develop a generic model that would incorporate the sixth order magneto-elastic potential and still explain the monostable and bistable approximations of the oscillator. With the help of the developed expressions, the dependence of magneto-elastic potential on parameters such as the dimensions and field strength of the magnets, the distance between the magnets, etc. is illustrated. Parametric studies have been conducted on the oscillator to elucidate the bifurcations occurring in the static equilibrium positions of the oscillator. Subsequently, the information pertaining to the bifurcations has been used to track the change in configuration of the oscillator from monostable to tristable, tristable to bistable, etc. Numerical simulations are also performed to characterize the free and forced vibrations in different configurations of the oscillator. The developed model is validated through experiments based on forced vibrations of the magneto-elastic oscillator. For a certain choice of excitation amplitude and frequency, the experimental response obtained in the case of the bistable and tristable configurations oscillates between the multiple stable states in a non-resonant and chaotic manner. This is in accordance with the experimental results reported in the literature.

The remainder of the manuscript is arranged as follows. Section 2 begins with a brief description about the magneto-elastic oscillator and develops a mathematical model for the same. The closed form expression for the nonlinear magneto-elastic potential is established in this section. Section 3 deals with the variation in the static equilibrium positions of the oscillator and consequently, its configuration, with respect to parameters such as magnetic field strength and distance between the magnets. The developed mathematical model is analyzed through numerical simulations and the results are presented in Section 4. Experimental validation of the developed model is presented in Section 5. Conclusions are drawn thereafter.

2. Magneto-elastic oscillator: Description and modeling

In this section, a brief description about the magneto-elastic oscillator is presented and subsequently, the governing equation of motion is developed. The oscillator consists of a vertical cantilever beam made up of a ferromagnetic material, carrying a permanent magnet at its tip. The beam bends between two permanent magnets under the combined influence of their magnetic field and the exogenous excitation (as shown in Fig. 1). This system is highly nonlinear and exhibits chaotic responses [2].

Fig. 1 shows the beam as a vertical cantilever of length L with harmonic base excitation $\ddot{x}_g = x \sin \omega t$. The beam carries a permanent magnet of radius a_t, length $2b_t$, mass w_t, and moment of inertia I_t at its tip. The transverse displacement at the tip mass is denoted as v. s represents the distance from the base to point P, along the neutral axis of the beam. The beam has