ARTICLE IN PRESS

Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Stability of dynamic response of suspension bridges

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ARTICLE INFO

Article history:
Received 4 June 2016
Received in revised form
31 December 2016
Accepted 6 January 2017
Handling Editor: W. Lacarbonara

Keywords:
Suspension bridges
Non-linear dynamics
Aeroelasticity
Stability
Parametric resonance
Floquet theory

ABSTRACT

The potential occurrence of internal parametric resonance phenomena has been recently indicated as a potential contributory cause of the appearance of critical dynamic states in long-span suspension bridges. At the same time, suspension bridges, in view of their flexibility, are prone to aeroelastic response, such as vortex shedding, torsional divergence and flutter. In this paper, a non-linear dynamic model of a suspension bridge is devised, with the purpose of providing a first attempt toward a unified framework for the study of aeroelastic and internal resonance instabilities. Inspired by the pioneering work of Herrmann and Hauger, the analyses have been based on a linearized formulation that is able to represent the main structural non-linear effects and the coupling given by aerodynamic forces. The results confirm that the interaction between aeroelastic effects and non-linear internal resonance leads to unstable conditions for wind speeds which can be lower than the critical threshold for standard aeroelastic predictions.

1. Introduction

Suspension bridges represent a spontaneous answer to demands of large spans, lightweight, high strength, ease of construction and aesthetic appearance. On one hand, the flexibility caused by the cable system and by the long span make the suspension bridges sensitive to dynamic loads; on the other hand, the relatively simple geometry of cable structures makes continuum approaches still very attractive, since can be based on a minimal number of non-dimensional parameters.

Early attempts to address the static equilibrium of suspension bridges were made by Moisseiff, who extended the elastic theory to the well-established Deflection Theory [1,2] by enforcing equilibrium in the deformed position, and accounting for the stiffening effect in the main cables. Earliest continuum models for the linear vertical vibrations of suspension bridges reproduced the effects of the stiffening truss girder by means of a Euler–Bernoulli beam supported by the main cables through inextensible and distributed vertical hangers. In this regard, the classic continuum model for the linear vertical vibration of suspension bridges, based on the so called linearized deflection theory, was first proposed by Bleich et al. [3], and Steinman [4], who derived some formulas for computing natural frequencies and mode shapes, and recently reviewed by Luco and Turmo [5]. The latter authors showed that the linear vibration of the considered suspension bridge model is completely governed by two non-dimensional parameters: the classic Irvine parameter of suspended cables, first introduced by Irvine [6], and a second parameter accounting for the relative stiffness of the girder with respect to the main cable system. Abdel-Ghaffar in the late 1970s [7–9] developed the methodology of free vertical, torsional and lateral vibration analysis of suspension bridges by means of a variational principle and a finite element approach. Then, the same author

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http://dx.doi.org/10.1016/j.jsv.2017.01.009
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[10–12] extended the continuum formulation to include coupling between vertical–torsional vibrations, nonlinear effects occurring in the case of large vibrations and the effects of distortional deformation of the girder cross-section.

Nowadays, in the design of suspension bridges a comprehensive set of wind related responses are taken into consideration, such as static divergence, vortex-shedding, buffeting and flutter. Indeed, the risk of developing aeroelastic instabilities is always present in lightweight long-span structures, characterized by high flexibility, low bending-to-torsional stiffness ratio and high width-to-depth ratio. Although such phenomena were already well known in aviation, aeroelastic effects did not represent an important issue in bridge design before the collapse of the Tacoma Narrows Bridge (USA) in 1940. Such a catastrophe was seen mainly as a direct consequence of flutter [13] that developed on the bridge deck at wind speed much lower than the design one. Flutter is generally studied within linearized aeroelastic models, which can provide the range of wind speeds where Hopf bifurcation occurs. To consider the effects due to the unsteadiness of the relative motion between the section and the air flow, indicial Theodorsen type [14–16] formulations can be adopted to predict more accurately the critical wind speed at the onset of the flutter instability [17] with respect to the quasi-steady formulation. The equations of motion for suspension bridges were employed for aeroelastic investigations in [18], where analysis were centered on experimentally determined flutter derivatives, and a full three-dimensional modal analysis of the structure.

It is well known from non-linear dynamics that, between coupled oscillators, energy transfer [19,20] can occur as far as the energetic levels reaches well-established critical thresholds. Classically, this behaviour is referred to as the *internal resonance* phenomenon. Many authors applied these principles to study the vibrations response of suspension bridges. The authors of [21,22] used the continuous model proposed by Abdel-Ghaffar [11], and solved the system of equations by means of the multiple scale perturbative technique [23]. Recently, Arioli and Gazzola [24], trying to explain why torsional oscillations suddenly appeared before the Tacoma Narrows collapse, found out that, also in isolated systems, vertical oscillations may switch to torsional ones, as long as they become large enough. The problem was already tackled by other authors [25–29] but it seems that there is still an open issue regarding the complete explanation of the sudden appearance of large oscillations which led to collapse. Hence, they paved the way for future works concerning the interaction between internal resonance and aeroelastic phenomena, as the present paper wants to do.

Indeed, the present article intends to study the stability of a suspension bridge model, following the preliminary attempt described in [27]. The analysis will exploit the continuum formulation of Abdel-Ghaffar [11], enriched by the aeroelastic actions coming from Theodorsen [16] indicial formulation for the wind-structure interaction. The stability will be checked in Lyapunov asymptotic sense, exploiting the well-known Floquet theory [30]. The variational system of equations is obtained following the pioneering procedure proposed by Herrman and Hauger [31], who assumed small but finite flexural perturbations coming from vortex-shedding excitation. The possibility of parametric internal resonances such as harmonic, sub-harmonic and super-harmonic, or additive combinational and anti-resonances will be checked by means of suitable stability maps.

2. The structural model

A continuum model of single span, linearly elastic, suspension bridge is considered (Fig. 1). The bridge, having a span length l, is composed of two main cables that support the stiffening girder (bridge deck) through uniformly distributed, massless and inextensible vertical hangers. The main cables are hinged at fixed anchors placed at the same vertical elevation and are modeled as mono-dimensional elements with negligible flexural, torsional and shear rigidities. The stiffening girder is modeled as an equivalent, uniform, Euler-Bernoulli beam, with flexural hinges and torsional forks at its ends. The distortional deformation of the cross-section is neglected. The cross-section of the girder is symmetric with respect to vertical local axis y (Fig. 1). The contribution of the stiffening girder in carrying dead loads is disregarded: dead loads are entirely carried by the main cables and are assumed to be uniformly distributed along the longitudinal axis.

2.1. Two-field formulation

Assuming zero horizontal displacement for the bridge deck, as well as zero transversal displacement and negligible longitudinal displacement for the cables, the motion of the bridge is described by means of two displacement functions (Fig. 2): vertical deflection $w_d(x, t)$ and twist rotation $\vartheta_d(x, t)$ around the longitudinal centerline of the deck, t denoting time. The equations of motion are derived by means of Hamilton's principle $\delta \int_{t_1}^{t_2} (T - V + W) dt = 0$, where, T and V are the total

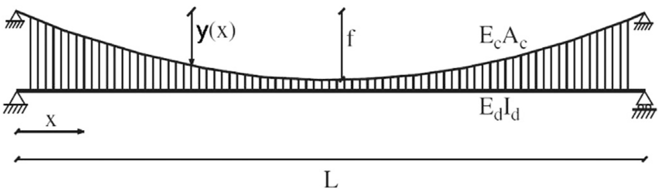

Fig. 1. Single span suspension bridge model.