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Nonlinear vibration analysis of axially moving strings based on gyroscopic modes decoupling

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ABSTRACT

A novel idea that applies the multiple scale analysis to a discretized decoupled system of gyroscopic continua is introduced and an axial moving string is treated as an example. First, the invariant manifold method is applied to the discretized ordinary differential equations of the axially moving string. Complex gyroscopic mode functions that agree well with true analytical results are obtained. The gyroscopic modes are subsequently used for the discretized ordinary differential equations with gyroscopic and nonlinear coupling terms that yield a gyroscopically decoupled system. Further the method of multiple scales is used to obtain the equations at a slow scale. This novel procedure is compared to solutions obtained by directly applying the classical multiple scale analysis to the gyroscopically coupled system without decoupling. The modal decoupled system analysis yields better frequency with comparing to the classic method. The proposed methodology provides a novel alternative for nonlinear dynamic analysis of gyroscopic continua.

1. Introduction

There exist wide applications of axially moving strings in many mechanical engineering systems such as serpentine belts, aerial cables, power transmission belts, magnetic tapes, textile fibers, etc. In contemporary engineering fields, the transverse vibration analysis of axially moving strings is generally known as a challenging work for safe design of a variety of machines and structural systems. There is an abundance of research papers on the analysis of transverse vibrations of axially moving string. The earlier investigations have been reviewed by Chen [1]. Recent developments of parametric excited and nonlinear vibrations of axially moving string may be referred to Kesimli et al. [2], Malookani, van Horssen [3] and the references therein.

Axially moving string is one of the simplest representatives of distributed gyroscopic systems. This system is of great interest to researchers due to its theoretical and practical importance in various industries. The distributed gyroscopic systems or gyroscopic continua involve complex vibration modes which lead to 'galloping nodes' during modal motions from experimental viewpoint [4,5]. For a Galerkin truncated system, the truncated finite dimension ordinary differential

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equations are coupled by the gyroscopic effects. Through discretization, the continuous system is projected to some finite orders of trial functions. The generalized coordinates for the trial functions are coupled by the gyroscopic effects and the functions can be treated further as linear or nonlinear systems. The static mode functions are usually adopted for axially moving materials and the efficiency of the Galerkin method has been verified [6–9] in free and forced vibration studies especially for dynamics in the sub-critical range. Recently, Malookani and van Horssen [10] demonstrated that the Galerkin method was invalid for long timescales when treating the parametric resonance of axially moving strings. The conclusion was an unusual case comparing with the classical results.

Due to gyroscopic effect caused by axially moving speed, the modal transverse vibration is not in an ‘in-unison’ sense, ie. the displacements of the whole string reach maximum values at the same time and go through equilibrium points at the same time. Rosenberg [11] defined ‘vibration in-unison’ as nonlinear normal modes which expanded the modal motions from linear undamped vibration to nonlinear damped vibration for non-gyroscopic systems. By introducing an invariant manifold, Shaw and Pierre [12,13] and Boivin et al. [14,15] expanded the nonlinear normal modes further which described the modal motions of the gyroscopic systems. Because the gyroscopic continuous or discretized systems in modal motions do not vibrate in unison, Hill et al. [16] introduced the term ‘out-of-unison vibration’ instead of normal modes because the gyroscopic modes are not normal in the classic point of view.

In this study, the gyroscopic complex modes based on the Galerkin truncated system is derived by the concept of invariant manifold introduced by Shaw and Pierre [12,13]. This discretization method is compared with the original continuous system to validate its efficiency. Further, the gyroscopic modes are applied to the nonlinear truncated system to obtain a decoupled system. The method of multiple scales is subsequently applied to investigate the 1:3 internal resonance based on the gyroscopic decoupled nonlinear system. The classic procedure is apply the multiple scale method directly to the gyroscopic coupled system. The results of the multiple scale analysis to both classical, gyroscopic coupled system and the decoupled system introduced in this paper are further compared and discussed with a numerical example. The novel procedure of multiple scale analysis for decoupled systems developed here and applied to axially moving strings can also be extended to other gyroscopic continua.

2. Governing equations and gyroscopic modal analysis

A string with length L and axial tension P_0 moves along the axis with axial velocity V between two fixed ends is considered. The partial differential equation that governs the nonlinear transverse vibration of an axially moving elastic string can be expressed as [1,17]

$$\rho W_{,TT} + 2\rho V W_{,XT} + (\rho V^2 - P_0) W_{,XX} = \frac{3}{2} E \left(W_{,X}\right)^2 W_{,XX}, \tag{1}$$

where W is the transverse defection of the axially moving string dependent on both the spatial variable X and temporal variable T, ρ is the linear density, and E is the Young's modulus of the string material. In Eq. (1), the comma-subscript notation denotes partial differentiation with respect to the variables after the comma. The following dimensionless variables and parameters are introduced

$$w = \frac{W}{L}, \quad x = \frac{X}{L}, \quad t = T\sqrt{P_0/\rho L^2}, \quad \gamma = V\sqrt{\rho/P_0}, \quad \kappa = E/P_0. \tag{2}$$

The governing partial differential equation in dimensionless terms is

$$w_{,tt} + 2\gamma w_{,xt} + (\gamma^2 - 1) w_{,xx} = \frac{3}{2}\kappa w_{,x}{}^2 w_{,xx}, \tag{3}$$

and the boundary conditions are

$$w(0, t) = w(1, t) = 0. \tag{4}$$

Substituting $w(x, t) = \varphi(x)e^{i\omega t}$ into a linearized system of Eq. (3) yields a second order ordinary differential equation with respect to x. Applying the boundary conditions (4), the complex modes and natural frequencies of the continuous, linearized system can be obtained as [4,18,19]

$$\varphi(x) = C\ \sin\left(k\pi x\right) e^{ik\pi\gamma x}, \quad \omega = k\pi\left(1 - \gamma^2\right), \quad k = 1,\ 2,\ 3,\ \ldots \tag{5}$$

The mode function in Eq. (5) is a complex function and it implies a traveling wave. For vanishing velocity, ie. $\gamma = 0$, the mode function degenerates to a static string.

The Galerkin truncation method and subsequently the invariant manifold method are applied to derive a discretized system. The complex modes obtained will be compared with the exact solutions (5) in order to verify the efficiency of the Galerkin method.

The solutions to the partial differential Eq. (3) can be expressed as

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