ARTICLE IN PRESS

Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

A semi-analytical beam model for the vibration of railway tracks

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ARTICLE INFO

Article history:
Received 23 May 2016
Received in revised form
6 November 2016
Accepted 20 December 2016
Handling Editor: M.P. Cartmell

Keywords:
Vertical/lateral coupling
Dispersion curve
Vignole rail
60E1
Frequency-wavenumber domain

ABSTRACT

The high frequency dynamic behaviour of railway tracks, in both vertical and lateral directions, strongly affects the generation of rolling noise as well as other phenomena such as rail corrugation. An improved semi-analytical model of a beam on an elastic foundation is introduced that accounts for the coupling of the vertical and lateral vibration. The model includes the effects of cross-section asymmetry, shear deformation, rotational inertia and restrained warping. Consideration is given to the fact that the loads at the rail head, as well as those exerted by the railpads at the rail foot, may not act through the centroid of the section. The response is evaluated for a harmonic load and the solution is obtained in the wavenumber domain. Results are presented as dispersion curves for free and supported rails and are validated with the aid of a Finite Element (FE) and a waveguide finite element (WFE) model. Closed form expressions are derived for the forced response, and validated against the WFE model. Track mobilities and decay rates are presented to assess the potential implications for rolling noise and the influence of the various sources of vertical-lateral coupling. Comparison is also made with measured data. Overall, the model presented performs very well, especially for the lateral vibration, although it does not contain the high frequency cross-section deformation modes. The most significant effects on the response are shown to be the inclusion of torsion and foundation eccentricity, which mainly affect the lateral response.

1. Introduction

The dynamic behaviour of railway tracks at high frequencies and their sound radiation characteristics are particularly important for the generation of rolling noise as well as other phenomena such as rail corrugation. The track vibration in both vertical and lateral directions contributes to the radiated noise. The excitation due to the surface roughness is vertical. However, as pointed out by Vincent et al. [1], the relative contribution of vertical and lateral components relies mainly on the location of the contact between the wheel and rail and on the attenuation of the respective waves along the rail. Typically the attenuation of the lateral waves is lower, and if there is a significant offset of the contact, the sound power due to lateral vibration may reach and even exceed the vertical component. Thus it is important to account for the rail radiation due to lateral/torsional waves in the rail. Many authors use analytical or semi-analytical models and focus mainly on the vertical track vibration but the lateral vibration, and especially the coupling of the vertical and lateral directions, have received much less attention [2].

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http://dx.doi.org/10.1016/j.jsv.2016.12.033

By comparing experimental results of the lateral track receptance (displacement due to a unit force) with the results of vertical track dynamic models, Grassie et al. [3] suggested that the latter could be adapted to represent the lateral response. For stiffer railpads (on wooden sleepers) good agreement was obtained between the measurements and a dynamic model of a beam on an elastic foundation, whereas for softer railpads (modern track with concrete sleepers) it was found that the rail head undergoes large lateral bending vibration, independent of that of the foot, the web itself acting as an elastic foundation. They thus used a two-layer continuous foundation model accounting for the rail head (as a Timoshenko beam) connected by a series of springs to the rail foot (continuous mass) and further connected to the second elastic layer representing the pads. Good agreement was found between this model and measurements of the track lateral receptance.

In the works of Thompson et al. [4–6], both vertical and lateral track behaviour was considered using a Timoshenko beam on a two-layer foundation model. This led to an under-prediction of the lateral receptance due to the neglect of torsion. An empirical parameter was introduced (X) to allow for the cross-coupling between vertical and lateral directions. The cross receptance (receptance in one direction due to a unit force in another direction) was estimated from the geometrical average of the vertical (A_y) and lateral (A_z) receptances ($A_{yz} = X\sqrt{A_y A_z}$), where X was obtained by comparison with measurements.

Wu and Thompson [7,8] developed continuously supported multiple beam models for both vertical and lateral vibration, which included cross-sectional deformation in an approximate way. Although they showed excellent agreement with a finite element (FE) model in terms of dispersion characteristics (wavenumber plotted against frequency), the effect of vertical-lateral coupling was not considered.

Thompson [9] obtained the dispersion relationship and receptance of an infinite rail based on the periodic structure theory (PST) of Mead [10]. This technique takes advantage of the fact that the cross-section remains constant along the rail and that its length is infinite. The structure is considered as a periodic structure with arbitrary period. A slice of rail 10 mm long was discretised by finite elements (FE), and the sleepers were also included in an equivalent continuously supported model. In this method, a commercial finite element software can be used to obtain the FE matrices.

The finite strip method was used by Knothe et al. [11], where only the cross-section of the rail was discretised and the elements were considered as infinite strips. The main advantages of this method over classical Finite Element Analysis are the reduced number of degrees of freedom, thus decreasing computational requirements and the avoidance of truncation effects at the end of the finite section of rail. Similarly, Gavrić [12] introduced the waveguide finite element (WFE) method for modelling of rails. In this method, as with the PST method, the infinite extent of the rail and the constant cross-section are taken into account. The cross-section is modelled using special two-dimensional finite elements, similar to the finite strip method. The displacement field across the cross-section is discretised by finite elements while complex exponentials are used to describe the waves in the longitudinal direction. Results were presented in the form of the dispersion of free waves propagating in an unsupported rail. Using the same approach, Ryue et al. [13] determined the waves propagating in a supported rail up to 80 kHz. Nilsson et al. [14] used the waveguide finite element method to calculate the vibration of an infinite, continuously supported rail excited by a point force. The forced response was also obtained by Gry [15] for a rail with periodic supports.

Bhaskar et al. [16] developed an analytical model accounting for the lateral and rotational motion of the rail. The frequency range of interest in this work was up to about 2000 Hz, thus it was considered important to account for the cross-sectional deformation occurring above 1500 Hz. The authors based their model on a finite element model developed by Ripke and Knothe [17] where the rail section is composed by three separate parts, representing the rail head as a beam in bending and torsion, and the rail web and foot by three plates (one for the web and one for the foot on each side of the web). A variational method was then used to obtain a model for an infinite rail continuously supported on railpads, sleepers and ballast in order to obtain the dispersion relationship and receptance. The rail head was allowed to translate in vertical, axial and lateral directions, as well as to rotate around the axial direction. The plates were allowed displacement in plane as well as deformation perpendicular to their plane. The railpads were represented by two springs set a distance apart equal to the width of the foot divided by $\sqrt{3}$ to account for the torsional stiffness as well. The responses of the rail were also obtained by means of Fourier integrals. A good agreement was found with the discretely supported finite element model developed by Ripke and Knothe [17], with the main differences occurring due to the continuous nature of the support.

In an attempt to understand and quantify the vertical-lateral coupling of rails, Betgen et al. [18] analysed the track mobility and decay rates by means of measurements and a Finite Element model and these were compared with the analytical models of Thompson et al. [6]. The rail was excited vertically at various locations across the top of the railhead and laterally at the side of the railhead. It was shown that the simplified Timoshenko beam models fail to capture many important characteristics of the response and that the cross mobility is significantly affected by the lateral position of the vertical force. The influence of asymmetry of the sleeper was found to be minimal, while the value of 0.3 (-10 dB) typically used for the factor X in the TWINS model was found to give reasonably good results for a lateral offset of 20 mm.

The main disadvantage of these numerical methods is that they require more extensive computational capacity and increased calculation times compared with analytical or semi-analytical models. Thus an analytical approach could be of great benefit, depending on the required level of accuracy and application. Moreover, an analytical model has the advantage of offering increased physical insight. In the present work, a semi-analytical approach is used to consider the various sources of vertical/lateral interaction. Cross-section deformation is not taken into account but instead the rail is treated as a simple beam cross-section, accounting for vertical and lateral bending, extension and torsion. Corrections for shear deformation, shear centre eccentricity and warping are included. The dispersion relationship is compared with results from a WFE model. The potential implications for rolling noise of the various sources of vertical/lateral coupling are presented in terms of the