ARTICLE IN PRESS

Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Flexural-torsional vibration of a tapered C-section beam

Scott T. Dennis*, Keith W. Jones

Department of Engineering Mechanics, United States Air Force Academy, CO 80840, United States

ARTICLE INFO

Article history:
Received 9 April 2016
Received in revised form
11 November 2016
Accepted 10 January 2017
Handling Editor: A.V. Metrikine

Keywords:
Beam
Taper
Vlasov
Thin-wall
C-section
Vibration

ABSTRACT

Previous studies have shown that numerical models of tapered thin-walled C-section beams based on a stepped or piecewise prismatic beam approximation are inaccurate regardless of the number of elements assumed in the discretization. Andrade recently addressed this problem by extending Vlasov beam theory to a tapered geometry resulting in new terms that vanish for the uniform beam. (See One-Dimensional Models for the Spatial Behaviour of Tapered Thin-Walled Bars with Open Cross-Sections: Static, Dynamic and Buckling Analyses, PhD Thesis, University of Coimbra, Portugal, 2012, https://estudogeral.sib.uc.pt) In this paper, we model the coupled bending-twisting vibration of a cantilevered tapered thin-walled C-section using a Galerkin approximation of Andrade's beam equations resulting in an 8-degree-of-freedom beam element. Experimental natural frequencies and mode shapes for 3 prismatic and 2 tapered channel beams are compared to model predictions. In addition, comparisons are made to detailed shell finite element models and exact solutions for the uniform beams to confirm the validity of the approach. Comparisons to the incorrect stepped model are also presented.

Published by Elsevier Ltd.

1. Introduction

Thin-walled beam geometries of open cross-section are prevalently used as structural members in weight sensitive aerospace, civil, and mechanical applications. For a beam cross-section with only a single plane of symmetry, the center of mass and shear center are offset giving coupled motion. Exact solutions for the natural frequencies and mode shapes for uniform beams that exhibit bending-torsion coupling are given by Dokumaci, Bishop et al., and Bercin and Tanaka. Dokumaci [1] solves a sixth order system based on Bernoulli-Euler bending and St. Venant torsion. Bishop, Cannon, and Miao [2] extended the Dokumaci solution to include the additional cross-section warping term for nonuniform torsion. The additional term results in an eighth order characteristic equation and the importance of the warping stiffness is apparent for thin wall open cross-sections. Shear deformation and rotatory inertia are included by Bercin and Tanaka [3] leading also to an eighth order system. For higher frequencies and thicker beam geometries, these additional terms become more influential. In the above cases, bending in a perpendicular direction is decoupled from the bending-torsion motion. If a uniform beam has no cross-sectional symmetry, bending motion in two perpendicular directions is coupled to the torsional motion. Tanaka and Bercin [4] present an exact 12th order system for the beam cross section with no symmetry. Bernoulli-Euler bending in two directions is coupled to nonuniform torsion.

Approximate solutions have been successfully attempted for the uniform thin-walled beam, e.g., Refs. [5–10]. Gere and Lin [5] determined the coupled frequencies for uniform thin-walled beams of open cross-section for various boundary

* Corresponding author.
E-mail address: std1979@gmail.com (S.T. Dennis).

http://dx.doi.org/10.1016/j.jsv.2017.01.017
0022-460X/Published by Elsevier Ltd.

conditions based on a Rayleigh-Ritz solution using the uncoupled mode shapes to approximate the coupled response. They define "triple coupling" for the general case of bending vibration in two perpendicular directions coupled to torsional vibration. Bernoulli-Euler bending and nonuniform torsion are assumed. For the more simple case where the beam cross section has an axis of symmetry, "double coupling" results where one of the bending vibrations is decoupled from the other motion. Mei [6] developed a finite element solution to the double coupled vibrating beam. Bernoulli-Euler bending and non-uniform torsion is modelled using an eight degree of freedom beam element. Rao and Carnegie [7] use Galerkin's method to solve for the natural frequencies of a cantilevered beam with triple coupling. Bernoulli-Euler bending and St Venant torsion is assumed. Bishop, Price, and Zhang [8] develop the equations of motion for a triple coupled Vlasov beam. They explain some of the differences between the Vlasov theory and the "modified Timoshenko theory" for the free vibrations of a double coupled beam. Tanaka and Bercin [9] present a coupled bending-torsion beam finite element approach to model Vlasov beam theory. The Vlasov beam differential equations retain warping stiffness, warping inertia, and rotatory inertia terms. The beam has 12 degrees of freedom based on Hermitian interpolation for bending about two axes and twisting. Sapountzakis and Dourakopoulos [10] develop a boundary element that captures coupled bending-torsional vibrations assuming Timoshenko beam theory, i.e., transverse shear deformation, warping stiffness and inertia, and rotatory inertia.

Additionally, the dynamic stiffness method has been applied to coupled beam motion in [11–14]. Hallauer and Liu [11] derive the exact 6×6 dynamic stiffness matrix for a straight prismatic beam and natural frequencies of a cantilevered three-beam wing compare favorably to a finite element solution. Friberg [12] applied Vlasov beam theory to a beam with an asymmetric open thin-walled cross section. The differential equations allow axial, bending, and twisting motion. Cross-section warping stiffness and rotatory/warping inertia terms due to Vlasov are retained although transverse shear flexibility is not. Banerjee and Fisher [13] include the influence of an axial force on the double coupled bending torsion natural frequencies of a cantilevered beam. Shear deformation, rotatory/warping inertia, and warping stiffness are considered negligible in this analysis. Further, Banerjee and Su [14] develop the dynamic stiffness matrix for triple coupled bending-torsion vibrations assuming Bernoulli-Euler bending and St Venant torsion. Numerical solutions are provided for double and triple coupled applications.

Klausbruckner and Pryputniewicz [15] use laser hologram interferometry to determine the natural frequencies and mode shapes for the coupled motion of uniform beams with channel shaped cross-sections. Timoshenko beam theory compares well to experiment for the shorter beam geometries. Higher modes are characterized by plate vibrations in the web and flanges and cannot be captured by the beam theory which assumes a rigid cross section.

Understanding beam motion for a tapered geometry has been more challenging; see [16–21]. Cywinski [16] summarizes extensive work on thin-walled members with variable open cross-sections. He found torsional natural frequencies for a tapered rod outside the range defined by the prismatic geometries, an apparent "paradox." Wekezer [17,18] reports similarly for a tapered I-shaped beam cross-section and warns that "stepped finite element models may lead to erroneous results." Rajasekaran [19] presents a thin-walled beam approach that includes geometric nonlinearity and found natural frequencies for a tapered beam agreeing with those of Wekezer. Nedelcu [20] presents the "generalized beam theory" based on Vlasov's equations but with the approximation that the warping displacements are still perpendicular to the plane of the cross-section for a tapered beam. He concludes the errors are acceptable for taper slope not greater than 5%. Shin et al. [21] state "meaningful errors" are incurred if stepped box beam elements are used to model a tapered box beam. They propose a 1D finite element that ensures displacement continuity of the box beam's thin plate walls and solutions compare well to tapered geometry solutions based on meshes of shell elements.

More recently, Andrade and colleagues [22,23] present a one dimensional approach for coupled stretching, bending, and torsion of tapered thin-wall beams with open cross sections, an extension of Vlasov's uniform beam theory. Thin-walled beams are modelled as "internally constrained membrane shells." This results in additional constraints and membrane strains unique to the beam with taper that contribute to the total torque. Andrade explains that regardless of the number of piecewise prismatic elements used to approximate a *tapered* beam, converged solutions to Vlasov's *uniform* beam theory are not correct. He recommends comparisons be made between his tapered beam approach and "the results of higher-dimensional finite element analyses (and, desirably, of experimental investigations as well)."

The purpose of this paper is to provide a numerical solution to the modified Vlasov equations for a tapered beam due to Andrade and to validate the theory and numerical solution by comparison to higher-dimensional finite element analysis and experimental measurements. The numerical solution is needed as an alternative to solutions derived from detailed and perhaps expensive shell finite element models. A finite element approach is taken where a new beam element is derived whose mass and stiffness matrices are found using the Galerkin approximation. The present numerical model assumes a piecewise prismatic, i.e., a "stepped" approach similar to aforementioned studies that have led to incorrect natural frequencies. Presently however the model includes the additional stiffness terms that are dependent on taper proposed by Andrade and colleagues. The natural frequencies and mode shapes for a C-section cantilevered beam are compared to experiment and Nastran analyses using shell elements. The Galerkin solution is first outlined. Details of the experimental approach for five beam geometries are next presented. Finally, experimental results from 3 prismatic and 2 tapered beams are compared to the Andrade theory.

Please cite this article as: S.T. Dennis, & K.W. Jones, Flexural-torsional vibration of a tapered C-section beam, *Journal of Sound and Vibration* (2017), http://dx.doi.org/10.1016/j.jsv.2017.01.017