ARTICLE IN PRESS

Journal of Sound and Vibration ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Nonlinear characteristics of an autoparametric vibration system

Zhimiao Yan [a,*], Haithem E. Taha [b], Ting Tan [a]

[a] School of Astronautics, Harbin Institute of Technology (HIT), Harbin, Heirongjiang 150001, China
[b] Department of Mechanical and Aerospace Engineering, University of California Irvine, Irvine, CA 92697-3975, USA

ARTICLE INFO

Article history:
Received 21 April 2016
Received in revised form
16 November 2016
Accepted 1 December 2016
Handling Editor: M.P. Cartmell

Keywords:
Passive control
Multiple actuation
Method of multiple scales
Saturation
Bifurcation
Chaos

ABSTRACT

The nonlinear characteristics of an autoparametric vibration system are investigated. This system consists of a base structure and a cantilever beam with a tip mass. The dynamic equations for the system are derived using the extended Hamilton's principle. The method of multiple scales (MMS) is used to determine an approximate analytical solution of the nonlinear governing equations and, hence, analyze the stability and bifurcation of the system. Compared with the numerical simulation, the first-order MMS is not sufficient. A Lagrangian-based approach is proposed to perform a second-order analysis, which is applicable to a large class of nonlinear systems. The effects of the amplitude and frequency of the external force, damping and frequency of the attached cantilever beam, and the tip mass on the nonlinear responses of the autoparametric vibration system are determined. The results show that this system exhibits many interesting nonlinear phenomena including saturation, jumps, hysteresis and different kinds of bifurcations, such as saddle-node, supercritical pitchfork and subcritical pitchfork bifurcations. Power spectra, phase portraits and Poincare maps are employed to analyze the unstable behavior and the associated Hopf bifurcation and chaos. Depending on the application of such a system, its dynamical behaviors could be exploited or avoided.

1. Introduction

Autoparameteric vibration systems are characterized by nonlinear internal coupling that involves at least two modes. Such coupling results in energy transfer from one mode of the system to another. The secondary mode, which is unforced, draws the energy from the primary mode and undergoes sustained oscillation. The primary mode is, as such, suppressed. For engineering applications, an autoparameteric vibration system can be used to suppress the oscillatory motion of the primary mode and/or generate motions in unactuated directions through excitation of the second mode [1].

For control purposes, autoparameteric vibration systems can be designed to suppress vibrations resulting from resonant or near-resonant excitations or oscillations due to Hopf bifurcation. Examples of such systems include civil structures (e.g., buildings, offshore rigs, towers and bridges). Such structures could undergo large-amplitude oscillations when the main frequency of seismic, wave or wind forces is close to the natural frequency of the civil structures. Dynamic motions due to Hopf bifurcation include flutter of wings and bridges, and galloping of iced-transmission lines and towers. To reduce their motions, a secondary system can be added. Haxton and Barr [2] devised an autoparametric vibration absorber by attaching a cantilever beam with a tip mass to a base structure subjected to external forces. Their experimental validation showed that

* Corresponding author.
E-mail address: zhimiaoy@hit.edu.cn (Z. Yan).

http://dx.doi.org/10.1016/j.jsv.2016.12.003

such an absorber is very effective in suppressing the vibrations of the base structure. Cartmell and Roberts [3,4] designed one autoparametric vibration system based on two coupled cantilever beams. Their theoretical and experimental results [3] indicated substantial internal coupling between two cantilever beams. Then, they [4] derived four-mode interactions and employed the first-order method of multiple scale to show that the system can exhibit non-synchronous large-amplitude responses of the indirectly excited modes in additional to the well-known phenomenon of saturation. Recently, Georgiades et al. [5,6] revisited this two-coupled-beam autoparametric vibration systems and derived the comprehensive models. Cartmell et al. [7] revised the secondary system by attaching a bar with a spring instead of a cantilever beam. In Cartmell's system, the relative placement of the mass along the bar could be modified to obtain the optimal natural frequency of the secondary mode for controlling the motion of the base structure. Other designs for autoparametric vibration systems are based on attaching pendulums to base structures, such as spring-mass-damper system [8], beam-mass-pendulum system [9], driven-Froude-pendulum system [10], multiple-pendulum systems [11,12] and pendulum-magnetorheological-damper-nonlinear-spring system [13]. Dai and Singh [10] analyzed the dynamic behavior of the driven-Froude-pendulum system and showed that the system undergoes periodic and quasiperiodic oscillations, and even chaotic motions. Vyas et al. [11,12] suppressed a large-amplitude resonant response of the base structure over a wide band of excited frequencies by attaching multiple pendulums of different natural frequencies to a single-degree-of-freedom linear system. Kecik and Borowiec [13] found that the nonlinear spring can effectively suppress the motion of the base structure and shrink the domains of unstable and chaotic responses. In application, Nagasaka et al. [14] used pendulum absorbers to suppress the vibration of helicopter rotor blades.

To date, most of the research on autoparametric vibration systems has aimed at controlling the motion of the main structure. Yet, these systems can be employed to create multi-directional actuations. Alternatively, the energy from the oscillatory motion of the secondary mode can be harvested and accumulated to operate self-powered devices including microelectromechanical systems (MEMS) or actuators [15–17] and wireless sensors [18,19]. The harvested power can replace small batteries that have a finite life span or would require hard and expensive maintenance [20–22]. In return, harvesting energy would reduce the motion of the secondary system. Recently, Kecik and Borowiec [23] proposed an autoparametric system with a pendulum and a nonlinear oscillator which harvests energy from the motion of the pendulum.

To enable the design of autoparameteric vibration systems that can be used for control, energy harvesting or multi-directional actuation, we revisit the model of a base structure and a cantilever beam with a tip mass proposed by Haxton and Barr [2]. Although, this special kind of autoparametric vibration system has been done to show its potential to effectively control a base structure that is subjected to resonant or near-resonant excitation, its nonlinear aspects have not been investigated. In this work, we perform a detailed analysis to show the effects of different parameters, such as the amplitude and frequency of the external force, damping and frequency of the attached cantilever beam, and tip mass, on the nonlinear response of the system. The extended Hamilton's principle is introduced to derive the governing equations in Section 2. In Section 3, the governing equations of the response of the cantilever beam are derived. In Section 4, the method of multiple scales is used to obtain an approximate analytical solution to the derived nonlinear set of differential equations. Solutions to different approximating orders are compared and validated. These solutions are used to analyze the stability and bifurcation of the system. Four different types of generated motions are discussed in Section 5. The effects of different parameters on these motions and nonlinear responses of the system are determined and presented in Section 6. The conclusions are presented in Section 7.

2. Modeling of the autoparametric vibration absorber

The autoparametric absorber system consists of a base structure and a beam with a tip mass. The base structure of this system is subjected to an external force $F(t)$, as shown in Fig. 1. This structure undergoes a vertical displacement x_d and has a stiffness k_2 and damping coefficient c_2. One local coordinate system $(x-y)$ is chosen to be fixed on the top of the base structure. The horizontal motion of the cantilever beam is denoted by $y(s)$, where s represents a curvilinear coordinate along the cantilever beam.

To establish the governing equation, we use the extended Hamilton's principle [24] which is written as

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W_{nc})\mathrm{d}t = 0 \tag{1}$$

where, T, V and W_{nc} are respectively the kinetic energy, potential energy and virtual work due to the nonconservative forces. The kinetic energy T and potential energy V are respectively expressed as