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Numerical results on noise-induced dynamics in the subthreshold regime for thermoacoustic systems

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ABSTRACT

Thermoacoustic instability is a serious issue in practical combustion systems. Such systems are inherently noisy, and hence the influence of noise on the dynamics of thermoacoustic instability is an aspect of practical importance. The present work is motivated by a recent report on the experimental observation of coherence resonance, or noise-induced coherence with a resonance-like dependence on the noise intensity as the system approaches the stability margin, for a prototypical premixed laminar flame combustor (Kabiraj et al., Phys. Rev. E, 4 (2015)). We numerically investigate representative thermoacoustic models for such noise-induced dynamics. Similar to the experiments, we study variation in system dynamics in response to variations in the noise intensity and in a critical control parameter as the systems approach their stability margins. The qualitative match identified between experimental results and observations in the representative models investigated here confirms that coherence resonance is a feature of thermoacoustic systems. We also extend the experimental results, which were limited to the case of subcritical Hopf bifurcation, to the case of supercritical Hopf bifurcation. We identify that the phenomenon has qualitative differences for the systems undergoing transition via subcritical and supercritical Hopf bifurcations. Two important practical implications are associated with the findings. Firstly, the increase in noise-induced coherence as the system approaches the onset of thermoacoustic instability can be considered as a precursor to the instability. Secondly, the dependence of noise-induced dynamics on the bifurcation type can be utilised to distinguish between subcritical and supercritical bifurcation prior to the onset of the instability.

1. Introduction

Thermoacoustic instability refers to self-sustained, large amplitude acoustic oscillations in confined combustors, such as employed in gas turbines, rocket motors and industrial burners. It occurs when a feedback loop forms between acoustic and heat release rate fluctuations, where the heat release rate fluctuations act as a source of energy to natural acoustic modes of the system geometry [1]. In combustion systems, thermoacoustic oscillations can lead to structural fatigue and even catastrophic failure [2]. It is, therefore, necessary either to avoid the operational regime under which the thermoacoustic instability occurs or to suppress the resulting oscillations. This requires the knowledge of stability margins of a system and

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the understanding of the dynamics of thermoacoustic instability, which are both influenced by interaction between thermoacoustic instability and other processes in a combustor [2–5].

The aspect of thermoacoustic instability that we investigate here deals with the interaction between thermoacoustic coupling and noise. As stochastic effects are dependent on the deterministic dynamics of the instability, discussions on noise-induced dynamics require a general overview of the dynamical nature of thermoacoustic instability. This is briefly presented in the paragraph below.

The onset of thermoacoustic instability, i.e. the transition from a state with no self-sustained oscillations to thermoacoustic oscillations, in a combustion system happens via a Hopf bifurcation. There are two variants of Hopf bifurcation—supercritical and subcritical—and both are observed in thermoacoustic systems [6–8]. Schematics of these two variants are shown in Fig. 1. It can be seen that in both variants of Hopf bifurcation, a stable focus, i.e. a non-oscillating system state, becomes unstable at a certain parameter value (x_H), known as Hopf point, after which the system undergoes transition to finite amplitude self-sustained oscillations, known as stable limit cycle. For supercritical Hopf bifurcation, self-sustained oscillations occur only after the Hopf point. While for subcritical Hopf bifurcation, self-sustained oscillations may occur even before the Hopf point in the regime between x_H and x_{SN}. The parameter value x_{SN} corresponds to the saddle-node point where the unstable and stable limit cycles meet. In this regime between x_H and x_{SN}, the system has two stable solutions, a stable focus and a stable limit-cycle, and hence is known as a bistable regime. In the regime before the Hopf point for supercritical Hopf bifurcation and before the saddle-node point for subcritical Hopf bifurcation, the only solution of the system is a stable focus, and therefore does not show any self-sustained oscillations. This regime is referred to as the *subthreshold* regime in this paper.

In practical systems, noise is always present and is mainly generated either directly by unsteadiness in combustion processes [9] or indirectly by flow turbulence and separation in combustion chamber [10,11]. Consequently, fluctuations in pressure of a combustor always contains noise-induced features regardless of the presence of thermoacoustic oscillations [12]. The noise-induced dynamics (i) can be used to obtain deterministic system parameters [13,14], (ii) can trigger thermoacoustic instability in the bistable regime [15–18], (iii) can change the stability margins of the system [19,20], and (iv) can act as a noisy precursor to instability [21,22].

In early studies on noise-induced dynamics (see Ref. [13] for an exhaustive survey), researchers were mainly interested in using the noise-induced dynamics to determine the stability margins of stable liquid rocket engines. This was done based on the analysis of statistical characteristics of the system's response to a large perturbation (e.g. via a small explosion inside the combustor). Towards similar goals, a theoretical model for the stochastic dynamics of thermoacoustic instability was proposed by Burnley and Culick [12]. This model was further employed by Seywert [13] to calculate the growth rate of the system prior to the Hopf point and statistical description of the thermoacoustic oscillations after the Hopf point in a Rijke tube model. This idea that noise-induced dynamics could be put to use for system identification was recently revisited by Noiray and Schuermans [14]. They extended the analysis to the identification of growth rates beyond the Hopf point, as well as the coefficient of nonlinearity that determines the amplitude of limit cycle oscillations. The methodology, which is based on the analysis of stochastic differential equations, was shown to be successful in extracting deterministic quantities underlying observed system dynamics in the case of a real engine.

Substantial previous research on the effects of noise has also been motivated to understand the noise-induced dynamics in the bistable regime of a combustion system undergoing thermoacoustic instability via a subcritical Hopf bifurcation. Burnley and Culick [15] have shown that as the system approaches the bistable zone, inclusion of stochastic terms leads to an evolution of the distribution of pressure amplitudes from a unimodal shape outside the bistable zone to a bimodal shape inside the bistable zone. They discussed how the repeated transition of the system between the upper branch of the

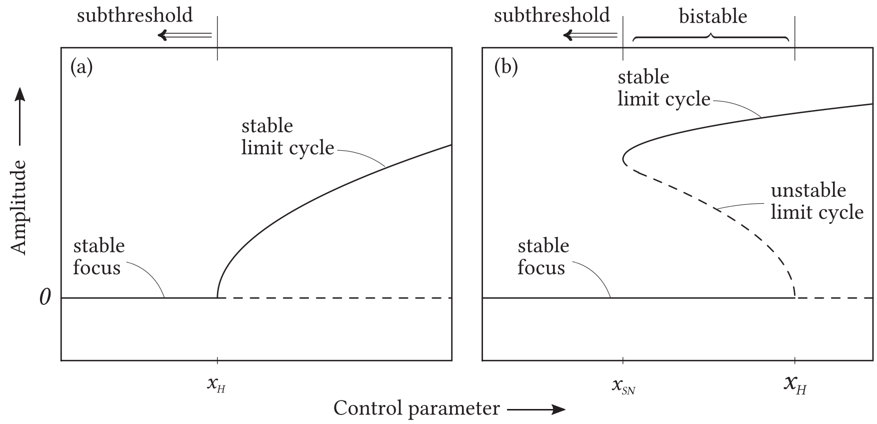

Fig. 1. (a) Supercritical and (b) Subcritical Hopf bifurcation plots. The bifurcation parameter values at the Hopf point and saddle-node point are marked as x_H and x_{SN}, respectively. The regime prior to x_H in (a) and prior to x_{SN} in (b) has only one stable non-oscillating solution (stable focus) and is referred to as subthreshold regime.

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