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Normal form analysis of a forced aeroelastic plate

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ABSTRACT

A nonlinear elastic plate in a supersonic unsteady flow forced by a dynamic excitation and a biaxial compressive load is studied. The physical behavior of the plate is modelized by the Von Kármán equations and the aerodynamic loads are modeled by using the piston theory including nonlinearities up to the third order. The space-continuum model is space-discretized by a Galerkin projection and then studied by a perturbation approach based on the Normal Form method in order to reduce the system to a simpler and essential form defined by its resonance conditions. A physical interpretation of the involved small divisors is given by analyzing how different equation parameters influence the reduced normal form model in the neighborhood of both static and dynamic bifurcation points.

1. Introduction

The general idea behind the Method of Normal Form, which was introduced by Poincareé [1], is to simplify a nonlinear system of ordinary differential equations in the neighborhood of an equilibrium point or periodic motion by introducing a suitable coordinate transformation (Ref. [2,3]). In particular, considering the system written as a linear (integrable) term perturbed by a nonlinear one, the introduced transformation brings the original system equation, in the new coordinates, as close as possible to a linear one (Ref. [4–6]). This theory, turns out to be a powerful device in the study of perturbed differential equations (see Ref. [2,7]) and can be considered as a geometric background of all the perturbation methods as Multiple Time Scaling or Averaging (see Ref. [5,6,8]). Moreover, it has been applied to the study of a large number of physical problems of practical interest, see Refs. [9–14]. The Normal Form transformation is related to the spectrum of the unperturbed system and can be defined only there are not combinations of the system eigenvalues which are zero (Refs. [2,4,15]). Otherwise, it can be demonstrated and shown that even if some eigenvalues combination, called divisors, are not zero but *small*, the obtained Normal Form perturbed solution is no longer valid and diverge from the exact solution (see Ref. [2,15]). The conditions for which some small divisor arises are more suitable and difficult to define, especially from the point of view of practical applications, with respect to the zero-divisors conditions. Moreover, both zero and small divisors, being related to combinations of system eigenvalues, are implicitly and intrinsically related to the perturbation form of the equations associated to the studied physical problem, *i.e.*, how its linear and nonlinear parts are defined. Such perturbation ordering is related to the physical hypotheses governing the observed physics and thus, it represents, roughly speaking, the link between the physical behavior of the system and its mathematical properties. For a deeper analysis of the small divisors phenomenon and their practical implications the reader can refer to Refs. [2,3,16] whereas in Ref. [17] a discussion on how

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the resonance conditions can influence the energetic distribution in the neighborhood of a Hopf bifurcation is given by using the Multiple Time Scaling together with a Proper Orthogonal Decomposition approach. In this paper, by using the Normal Form theory on a classical problem of Mechanics some relations between the small-divisors and physical parameters in the equations are shown and discussed giving a new insight into the concept of small divisors by a physical interpretation and a quantitatively definition of the term *small*. For this purpose, a Von Kármán plate forced by a vertical dynamic excitation, subject to a biaxial static load and invested by a supersonic flow has been considered. This physical model is able to represent a variety of response scenarios (see Refs. [18–22,17]) and it has been widely studied in literature, both considering its abstract mathematical properties (see Refs. [23–27]) both analytically and numerically analyzing its nonlinear behavior analytically as in Refs. [17,28,16,29,30].

The performed analysis is started by representing the fluid-structure interaction by using the Piston Theory (see Ref. [31]) and reducing, via a Galerkin projection on suitable functional basis, the partial differential equation model to a system of nonlinear forced and coupled oscillators of Duffing-type. The Normal Form method is then applied to the obtained system of coupled nonlinear oscillators in order to reduce it to a simpler form defined by resonance conditions. Finally, by analyzing how different physical parameters drive the resonance conditions, a physical interpretation of the involved small divisors is given and, in particular, the role of damping is emphasized in the spread of chaos together how the load can influence the nature of the obtained perturbed solutions. Moreover, the performed analyses provides useful informations for both static and dynamic cases about which parameters really influence the nonlinear behavior of the considered physical problems.

In Sections 2–4 some key issues about the studied problem, the space-discretization process and the Normal Form perturbation approach are discussed. In Section 5 several stability scenarios are presented together with a discussion of the obtained analytical and numerical results. Finally, in Section 6 concluding remarks are addressed.

2. Governing equations and boundary conditions

Let us consider the equilibrium of an elastic rectangular panel simply supported and exposed to a supersonic flow having velocity U. By employing the Kirchhoff-Love hypothesis for the nonlinear theory of elastic plates (see Refs. [18–20]) which assumes that the deflections of the shell are comparable with its thickness h, but small compared with its edge lengths a and b, the partial differential equations governing the considered physical problem are (see Ref. [18]):

$$\rho h w_{,\tau\tau} + \rho h \delta w_{,\tau} + D\nabla^4 w = [\phi, w] + f + \Delta p \tag{1}$$

$$[\phi, w] = \phi_{,yy} w_{,xx} + \phi_{,xx} w_{,yy} - 2\phi_{,xy} w_{,xy} \tag{2}$$

$$\Delta p = p - p_\infty \tag{3}$$

$$\frac{1}{Eh}\nabla^4\phi = (w_{,xy})^2 - w_{,xx}w_{,yy} \tag{4}$$

$$\nabla^4(\cdot) = (\cdot)_{,xxxx} + (\cdot)_{,yyyy} + 2(\cdot)_{,xxyy} \tag{5}$$

where the subindex indicates partial derivatives with respect to the variables x,y and t, $w(x, y, t)$ is the vertical displacement, D is the plate bending stiffness, δ is the damping coefficient, ρ is the material density of the panel, E the Young modulus, $p - p_\infty$ is the upper-lower side differential pressure load on the panel surface, f is an external pressure and ϕ the Airy's stress function defined by the following relations:

$$N_x = \phi_{,yy} \quad N_y = \phi_{,xx} \quad N_{xy} = -\phi_{,xy} \tag{6}$$

Eq. (6) implies that the in-plane equilibrium equations are automatically satisfied:

$$N_{x,x} + N_{xy_{,y}} = 0 \quad N_{yx_{,x}} + N_{y_{,y}} = 0 \quad N_{yx} = N_{xy} \tag{7}$$

The simply supported panel boundary conditions are:

$$w|_{x=0} = w_{xx}|_{x=0} = 0 \quad w|_{x=a} = w_{xx}|_{x=a} = 0 \tag{8}$$

$$w|_{y=0} = w_{yy}|_{y=0} = 0 \quad w|_{y=b} = w_{yy}|_{y=b} = 0 \tag{9}$$

whereas as initial conditions one can assume that:

$$w(x, y, 0) = \tilde{w}(x, y) \quad w_{,t}(x, y, 0) = \dot{\tilde{w}}(x, y) \tag{10}$$

The boundary conditions for Eq. (4) can be imposed on the average (see Ref. [18] for the details and a deeper analysis). In particular, it is possible to assume that