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Passive simulation of the nonlinear port-Hamiltonian modeling of a Rhodes Piano

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ABSTRACT

This paper deals with the time-domain simulation of an electro-mechanical piano: the Fender Rhodes. A simplified description of this multi-physical system is considered. It is composed of a hammer (nonlinear mechanical component), a cantilever beam (linear damped vibrating component) and a pickup (nonlinear magneto-electronic transducer). The approach is to propose a power-balanced formulation of the complete system, from which a guaranteed-passive simulation is derived to generate physically-based realistic sound synthesis.

Theses issues are addressed in four steps. First, a class of Port-Hamiltonian Systems is introduced: these input-to-output systems fulfill a power balance that can be decomposed into conservative, dissipative and source parts. Second, physical models are proposed for each component and are recast in the port-Hamiltonian formulation. In particular, a finite-dimensional model of the cantilever beam is derived, based on a standard modal decomposition applied to the Euler-Bernoulli model. Third, these systems are interconnected, providing a nonlinear finite-dimensional Port-Hamiltonian System of the piano. Fourth, a passive-guaranteed numerical method is proposed.

This method is built to preserve the power balance in the discrete-time domain, and more precisely, its decomposition structured into conservative, dissipative and source parts. Finally, simulations are performed for a set of physical parameters, based on empirical but realistic values. They provide a variety of audio signals which are perceptively relevant and qualitatively similar to some signals measured on a real instrument.

1. Introduction

Sound synthesis based on physical modeling aims at recovering natural behaviors of existing (or imaginary) instruments. This includes transients, effects due to damping phenomena, timbre variations due to nonlinearities, etc. However, since the models are nonlinear, guaranteeing numerical stability is not straightforward. In this context, approaches based on energy have been developed and applied to simulate musical instruments [1–4]. The principle relies on passivity: conservative (or

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dissipative) phenomena make the energy time-variation equal to (or lower than) the power received from external sources. Passivity provides energy bounds, from which state bounds stem, so that preserving this property in simulations can be used to address stability issues.

Most of these methods involve space-time discretizations and numerical schemes that are devoted to handle conservative problems and that can successfully be applied to more realistic dissipative models. Technically, these methods are usually designed in such a way that a numerical power balance is fulfilled for the discretized quantities (*e.g.* as the product of discrete velocities and forces). In this paper, this point of view is modified and handled in two steps: (I) derive a passive model in the continuous time-domain, through conservative interconnections of passive elementary components; (II) transpose the complete system in discrete-time domain in such a way that the original power balance is naturally fulfilled and passivity is naturally preserved.

Numerous methods are available to reach step (I). They can be divided in two main classes: (WS) *wave scattering methods* and (KV) *Kirchhoff's variables methods* (see *e.g.* [5]). Mixed WS/KV methods have also been proposed [6]. Several methods are also available for step (II). *Wave-digital filters* (WDF) [7] and *digital wave-guide* (DWG) [8] are commonly used in audio and acoustic applications. These formalisms belong to the class of WS methods. They allow block-based modeling approaches [6], by introducing links that mimic the serial and parallel connections, and result in passive models for linear systems [9]. WDF and DWG approaches lead to realizable and explicit numerical systems. They are appreciated in real-time sound synthesis applications. However, their benefits are lost for nonlinear systems.

This paper deals with a nonlinear system: the Fender Rhodes piano. Its passive modeling is derived in the class of Port-Hamiltonian Systems, introduced in the 1990s [10–12]. These systems can be considered as an extension of Hamiltonian systems [13] in the sense that these dynamical systems can be composed of conservative components. But they also can include dissipative components as well as some ports connected to external sources and through which energy can transit. These systems admit a power balance that can be decomposed into conservative, dissipative and source parts. They also can be simulated in such a way that the power balance (structured into conservative, dissipative and source parts) is preserved in the discrete-time domain, including for nonlinear systems, see [14], p. 32 and [15]. These modeling and simulation tools are chosen, adapted and used to address the sound synthesis of the Fender Rhodes piano. Preliminary results have been presented in [16].

The paper is organized as follows. In Section 2 the problem statement presents a simplified description of the Rhodes piano and sets the objectives. Section 3 introduces the port-Hamiltonian (pH) formulation. Section 4 is devoted to the physical modeling and the finite-dimensional pH formulation of elementary components. In particular, a finite-dimensional model of an Euler-Bernoulli cantilever beam is derived, based on a standard modal decomposition.

Then, in Section 5, elementary components are connected, yielding the nonlinear finite-dimensional port-Hamiltonian system to simulate (step I). Section 6 details the numerical method that preserves the power balance (step II). Finally, in Section 7, numerical results are presented and some signals are compared to a few measurements.

2. Problem statement

This section describes the Rhodes piano electromechanism. It focuses on the components that are selected to derive the physical model. Then, it states the scientific issues to be addressed to reach guaranteed-passive sound synthesis.

2.1. Overview and main components

A description of the Rhodes piano is given in Fig. 1. The complete system is quite complex (38 components mentioned for each note in Fig. 1ⓐ). The resonator is an asymmetric tuning fork (elements 7 to 13 and 19 in Fig. 1ⓑ), where one of the prong is call the tine (element 13) and is struck by the hammer (elements 14 and 15), the other prong is called the tone bar

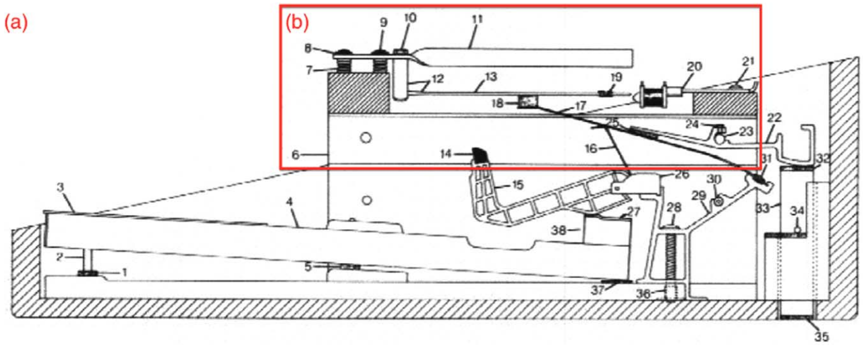

Fig. 1. Overview of a single note of a Rhodes piano: ⓐ original schematics [17] (extracted from http://www.fenderrhodes.com courtesy of Frederik Adlers); ⓑ part selected for modeling.

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