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System parameter identification from projection of inverse analysis

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ABSTRACT

The output of a system due to a change of its parameters is often approximated with the sensitivity matrix from the first order Taylor series. The system output can be measured in practice, but the perturbation in the system parameters is usually not available. Inverse sensitivity analysis can be adopted to estimate the unknown system parameter perturbation from the difference between the observation output data and corresponding analytical output data calculated from the original system model.

The inverse sensitivity analysis is re-visited in this paper with improvements based on the Principal Component Analysis on the analytical data calculated from the known system model. The identification equation is projected into a subspace of principal components of the system output, and the sensitivity of the inverse analysis is improved with an iterative model updating procedure. The proposed method is numerical validated with a planar truss structure and dynamic experiments with a seven-storey planar steel frame. Results show that it is robust to measurement noise, and the location and extent of stiffness perturbation can be identified with better accuracy compared with the conventional response sensitivity-based method.

1. Introduction

The system output due to a change in its parameters is often analyzed with partial derivatives in many applications. The sensitivity method may be an alternative to assess the system parameters with frequency domain data [1–7] and time domain data [8–15] in structural condition assessment.

The inverse dynamic analysis was proved to have converged solution [16] when both the system and excitation are not known. Law et al. [8] developed an acceleration response sensitivity-based method with the wavelet packet energy to identify the local stiffness change in a structure. The wavelet coefficient sensitivity was also derived for structural parameter perturbation assessment [9]. Lu and Law [10] studied features of the dynamic response sensitivity of a structure under different types of excitations. The sensitivity-based method was then employed to estimate local stiffness change and

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excitations applied [11,14]. Law and Ding [12] later used this method for substructural condition assessment, and another interface force sensitivity method [13,17] and power spectral density transmissibility method [15] were also proposed to identify local stiffness change in a substructure. The stiffness change of the structure in the above-mentioned studies can be identified via model updating as long as the vibration of the structure can be represented with a set of linear differential equations with nonlinear relationship between the observation data and the structural parameter.

The inverse sensitivity analysis for the identification of system parameter perturbation is re-visited in this paper. The relationship between the system parameters and its output has been broadly studied with a combination of sensitivity analysis and Principal Component Analysis (PCA). The sensitivity analysis in a functional output system has been shown [18] feasible when the outputs are expanded in a suitable functional coordinate system. Prendergast [19] has commented that the observation on system output can be seriously affected by the sensitivity of the principal component. The output variation of system was predicted basing on the principal components of changes of the system parameters [20]. The output due to a change in the system parameter of a linear system was then estimated by PCA with simultaneous computation of the sensitivities from the system output. Yamanishi and Tanaka [21] included the sensitivity analysis as the basic tool in their functional Principal Component Analysis. The PCAs of sensitivities were decomposed [22] in a study of the free energy of bio-molecular complexes.

The PCA decomposes a data series into principal components via orthogonal linear transformation, and the ones corresponding to larger eigenvalues of the covariance matrix of the data set contain more information on the variation of system parameter. The change in the system output was estimated from the principal component of changes in the system parameter [21], and from the principal component of the sensitivities [22]. The perturbation of system parameters was estimated from the principal components of the observation data which is, however, subjected to measurement noise. The above shows that the PCA has been used in the solution of linear sets of equations with sensitivity analysis [18,21]. More recently, Ni et al. [23] projected the frequency response functions (FRFs) into the subspace defined by the principal components of the measured FRFs to assess the seismic damage of structures. The PCA in this paper is for the projection of the sensitivity analysis and subsequent model updating of the system.

It has been shown feasible to project the sensitivity equation into a subspace that contains more information of the system with less noise effect [24]. Such projection can enhance the response sensitivity leading to better estimation results on the system parameter perturbation. The measured response was analyzed with singular spectrum analysis to get the trend components which define the corresponding projection subspace. The principal components corresponding to larger eigenvalues of the covariance matrix of the data set can be selected as representatives of the data set, and the system parameter perturbation can be estimated with projection of the sensitivity equation in the subspace.

This paper presents a generalized projection method with the analytical responses decomposed with PCA, and the sensitivity equation is projected in the subspace defined by the principal components instead of the measured responses to eliminate some of the noise effect. The projected sensitivity equation is normalized with the eigenvalue of the principal components, and the resulting equation is dominated with the influence of the eigenvectors (direction of projection) and less affected by the eigenvalues. Although this method adopts the principal components to construct the projection subspace, it is applicable to any kind of decomposition with orthogonal property.

The sensitivity analysis method and model updating procedure are briefly reviewed below, and the proposed method is then described in detail with simulation on the parametric system. The proposed method is then generalized for stiffness perturbation of the dynamic system and the selection of projection subspace is discussed. It is then validated with simulation studies on a dynamic system of a plane truss and laboratory experiment on the dynamic system of a seven-storey planar steel frame.

2. Methodology

The sensitivity analysis is integrated with the PCA and model updating in a new approach for an enhanced sensitivity with less noise effect in the updating procedure.

2.1. Existing sensitivity method

The sensitivity coefficient in an input-output problem defines the output gradient with respect to a change in the system parameter. The sensitivity method is briefly reviewed in this section.

The study is illustrated in this section with a set of second order differential equation as

$$f(\mathbf{y}(t,\boldsymbol{\theta}),\dot{\mathbf{y}}(t,\boldsymbol{\theta}),\ddot{\mathbf{y}}(t,\boldsymbol{\theta}),\boldsymbol{\theta},t)=0 \tag{1}$$

where $\mathbf{y}(t,\boldsymbol{\theta})$, $\dot{\mathbf{y}}(t,\boldsymbol{\theta})$, $\ddot{\mathbf{y}}(t,\boldsymbol{\theta})$ represent the system outputs, $\boldsymbol{\theta}$ represent the system parameters, t denotes the time instant, and f denotes a function.