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Iterative regularization method in generalized inverse beamforming

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ABSTRACT

Beamforming based on microphone array is a method to identify sound sources. It can visualize the sound field of the source plane and reveal interesting acoustic information. Generalized inverse beamforming (GIB) is one important branch of beamforming techniques due to its high identification accuracy and computational efficiency. However, in real testing situation, errors caused by measurement noise and configuration problems may seriously reduce the beamforming accuracy. As an inverse problem, the stability of GIB can be improved with regularization methods. We proposed a new iterative regularization method for GIB by iteratively redefining the form of regularization matrix and calculating the corresponding solution. Moreover, the new method is applied to functional beamforming and double-layer antenna beamforming respectively. Numerical simulations and experiments are implemented. The results show that the proposed regularization method leads to more robust beamforming output and higher accuracy in both the two applications.

1. Introduction

Beamforming is a technique used to process microphone array data in order to find the direction of incident acoustic waves and estimate the power of sound source at medium or high frequency [1]. The most common form of beamforming is the delay-and-sum beamforming (DAS) which compensates the time-delay between the signals received by the microphones in order to localize the source by constructive interference [2]. For enhancing the adaptability of beamforming method under different test conditions such as frequency, distance and array geometry, quantities of new methods have been proposed over the last two decades.

One of the most important branches is generalized inverse beamforming whose basic idea is to build an inverse problem model which represents the solution procedure of the beamforming output. The deconvolution method is proposed by Brooks [3]. It aims at identifying Point Spread Functions (PSF's) in source maps. The PSF's are theoretical beam patterns obtained by applying conventional beamforming using synthetical microphone data of monopole point sources. The objective of deconvolution methods is to replace these PSF's by single points, or beams with narrow widths. Among all kinds of deconvolution methods, DAMAS [3] and CLEAN [4] are the most widespread.

The generalized inverse beamforming (GIB) is another representative algorithm presented by Takao Suzuki. After

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decomposing the microphones Cross Spectral Matrix (CSM) into eigenmodes, an inverse problem is defined and solved iteratively [5]. Compared with conventional deconvolution approaches, the GIB has higher computational efficiency and similar identification accuracy.

However, when it comes to low SNR (Signal to Noise Ratio) situation, the accuracy and stability are not satisfying, which somehow limits the interpretation and relevance of beamforming maps [6]. In this paper, the accuracy is defined as the main lobe width of the source map. The stability indicates the inverse solution's sensitivity to the measurement noise and model uncertainties. It can be judged by discrete Picard condition [7], condition number [8] and so on [9]. This problem tends to be more obvious and significant in real test situations. The main reason for the problem is that the inverse solution without regularization is ill-conditioned [8], indicating that the solution can be very sensitive to measurement noise or model uncertainties. This is characteristic of inverse acoustical problems. The errors caused by test configuration and noise in measurement data may lead to large perturbation in output data [8].

To solve the ill-conditioned inverse problem, appropriate regularization method [10–12] should be applied. The most classic regularization method is Tikhonov regularization [13] and it is also suitable for the inverse problem in acoustic source localization with microphone array [14]. The regularization matrix and parameter are the key factors of the general Tikhonov solution of inverse problem. In contrast with regularization parameter, the regularization matrix also has significant effect on the accuracy and stability of beamforming output. That can be found through the general Tikhonov solution equation [9,15]. In conventional GIB, regularization parameter is selected by an empirical formula while the regularization matrix is simplified to an identity matrix. That is not enough for more accurate identification when the source frequency is not so high and the SNR is rather low (less than 15 dB).

Therefore, to obtain more accurate inverse beamforming output at low SNR, a new method combining generalized inverse beamforming with functional beamforming is proposed by Shu Li [16,17]. In his core algorithm, the regularization matrix is defined iteratively.

We have made further researches on Shu Li's iteration method. We found that the acoustical map of Shu's method is not accurate enough when increasing the dynamic range, especially when SNR is low. This may be related to the algorithm stability. To enhance the regularization strength and accurately detect the sound source, we propose a double iterative regularization method in this paper. This modified algorithm is also combined with two other beamforming methods, functional beamforming [18,19] proposed by Dougherty and double layer antenna beamforming [20,21] proposed by Pascal, respectively. Each application shows that this method can achieve great improvements in output accuracy and stability both in simulated and experimental data.

The remainder of this paper develops as follows: Section 2 introduces basic theory of the inverse beamforming and its regularization. Section 3 explains the modified beamforming method with regularization strategy. This is used in Section 4 for the improved high order matrix function based beamforming with simulated and experimental data. Section 5 illustrates another application of the proposed beamforming algorithm, followed by simulated and experimental data. Final conclusions are given in Section 6.

2. Inverse beamforming and its regularization

The direct source-receiver model is

$$p = \mathrm{Gq} \tag{1}$$

where $\mathbf{p}_{M\times 1}$ is the acoustic pressure measured by the microphones and the subscript M indicates the number of microphones, $\mathbf{G}_{M\times L}$ is the transfer matrix and the subscript L indicates the number of scan points, and $\mathbf{q}_{L\times 1}$ is the acoustic pressure in the source scanning plane, expressed as a vector.

From Eq. (1) we can find that by calculating the inverse of $\mathbf{G}$ we can directly solve $\mathbf{q}$ in ideal situation. However, due to the ill-conditioning of the problem, a small perturbation in measurement data or transfer matrix $\mathbf{G}$ may enormously reduce the accuracy of beamforming output. Therefore, regularization is needed.

The most classic regularization method, Tikhonov regularization, introduces a penalty function which consists of regularization matrix $\mathbf{L}$ and regularization parameter λ to obtain a more accurate solution of inverse problem.

An optimal inverse solution for Eq. (1) is found by solving this least-mean-square problem:

$$\mathbf{q}_\lambda = \arg\min\left\{ \|\mathbf{p} - \mathbf{G}\mathbf{q}\|_2^2 + \lambda^2 \boldsymbol{\Omega}(\mathbf{q})^2 \right\} \tag{2}$$

where $\|\cdot\|_2$ is 2-norm, $\boldsymbol{\Omega}(\mathbf{q}) = \|\mathbf{L}\mathbf{q}\|_2$, which is a smoothing norm of $\mathbf{q}$. $\mathbf{L}$ is the regularization matrix, it works with regularization parameter λ to reduce the illness of inverse problem [9].

As for regularization matrix $\mathbf{L}$, when $\mathbf{L}=\mathbf{I}$, Eq. (2) is rewritten as

$$\mathbf{q}_\lambda = \arg\min\left\{ \|\mathbf{p} - \mathbf{G}\mathbf{q}\|_2^2 + \lambda^2 \|\mathbf{q}\|_2^2 \right\} \tag{3}$$

By solving Eq. (3), one obtains the standard solution of inverse problem. For standard solution, the regularization matrix is simplified to an identity matrix and has little accurate control on the regularization of inverse problem, since the