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Long-term stable time integration scheme for dynamic analysis of planar geometrically exact Timoshenko beams

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ABSTRACT

In this paper, an energy-momentum method for geometrically exact Timoshenko-type beam is proposed. The classical time integration schemes in dynamics are known to exhibit instability in the non-linear regime. The so-called Timoshenko-type beam with the use of rotational degree of freedom leads to simpler strain relations and simpler expressions of the inertial terms as compared to the well known Bernoulli-type model. The treatment of the Bernoulli-model has been recently addressed by the authors. In this present work, we extend our approach of using the strain rates to define the strain fields to in-plane geometrically exact Timoshenko-type beams. The large rotational degrees of freedom are exactly computed. The well-known enhanced strain method is used to avoid locking phenomena. Conservation of energy, momentum and angular momentum is proved formally and numerically. The excellent performance of the formulation will be demonstrated through a range of examples.

1. Introduction

Dynamics of beams remains a very active research area. Its applications are found in many branches such as structural and aerospace engineering [1], multibody dynamics [2–4], nano and bio dynamics [5,6]. The description of the nonlinear kinematics of planar beam can be achieved using an inertial frame (Total Lagrangian approach) [7–9], or alternatively, via a floating frame [10,11] or a co-rotational approach [12–14]. The inertial frame is a more direct and natural approach for geometrically exact dynamics of the present beam model.

In many applications such as petroleum engineering [15] and multibody dynamics of flexible rod-type structures [2,4] among others, the members are slender beams, so an Euler-Bernoulli model could be completely sufficient. However, the model is characterized by a highly complex nonlinearity, because of the assumption that the vector normal to the centre line remains normal after the deformation [16–18]. Further, it exhibits a complex expression for the rotational inertia term. As an alternative, the Timoshenko-type beam proved to be a very popular choice. It still assumes that the cross section remains in a plane after the deformation but allows for shear deformations to take place [10,19–23]. The degrees of freedom are increased by adding rotational onee but the strain relations are simpler and, in a finite element context, it requires only C^0 continuity. Additionally, using Timoshenko-type of beam, the inclusion of the rotational inertia is more straightforward. Two possible choices of strain measures for Timoshenko-type of beam are available: the formulation based on the stretch-type

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strain measures in the spirit of Cosserat theories [24,25] and the one using metric-based strain measures such as the right Cauchy-Green deformation tensor which is used in this paper.

The linear interpolation functions used for Timoshenko-type of beam element can lead to numerical ill-conditioning resulting in e.g. shear locking phenomena. In the literature, there are already several methods to overcome this locking including reduced integration, mixed and enhanced strain methods [19,26–28]. With a view on possible future applications with a non-linear constitutive law, the enhanced strain method is favoured.

Beyond the description of the beam kinematics and the choice of the corresponding strain measures, much research has been carried out to develop an effective time integration scheme for non-linear dynamic problems [16,29–31]. Many implicit time integrators (the classical Newmark method, trapezoidal rule, midpoint rule) unconditionally stable in linear analysis [32], are not stable in the non-linear regime and suffer from severe shortcomings. One of the earliest approaches to achieve the stability in the non-linear regime is to introduce some numerical damping as in HHT-alpha method [33,31]. Its stability depends, however, on the selection of adjusted parameters which is not straightforward; the right choice of parameters depends on the outcome of the problem itself, otherwise inappropriate parameters can lead to large solution errors. Alternative time-stepping methods suggest to use constraints to enforce the energy and momenta conservation via Lagrange multipliers [34]. For nonlinear problems, this algorithm can lead to convergence difficulties of the iterative procedure. Over the last two decades, a considerable effort has been made towards the so-called energy-momentum conserving schemes. The algorithms are specially designed for undamped systems which inherently conserve both energy, linear and angular momentum. The earliest energy-momentum scheme, proposed by Simo and Tarrow [30], is only valid for quadratic non-linearities in the strain-displacement relation. Sansour et al. [35,36] developed successfully an energy-momentum method valid for any non-linear strain-displacement relation. As an extension of the work in [37], Leyendecket et al. [38] designed an objective energy-momentum conserving scheme for geometrically exact beams subjected to holonomic constraints. Bathe [29] also presented a method where the trapezoidal rule and the three-point backward Euler method are combined and applied to non-linear analysis with large deformations. A further discussion and analysis on the definition of the stress tensor at equilibrium for energy-momentum methods is given in [39]. Gautam and Sauer [40] recently explored an energy-momentum conserving scheme for adhesive contact problems. We refer to [41–44] for further examples of numerical treatments of relevant non-linear dynamical systems.

In this paper, we propose an energy-momentum method for the elastodynamics of in-plane geometrically exact Timoshenko-type beams, where the rotational degrees of freedom are exactly integrated. The outline of this paper is as follows. In the next section, the kinematics of the proposed geometrically exact Timoshenko-type beam model, along with the elastic constitutive law and the enhanced strain method, are presented. In Section 3, we derive the dynamic equations from Hamilton's principle and provide the field equations. Section 4 deals with finite element discretisation and the formulation of the time-stepping scheme which is at the heart of the paper. The formal proof of the energy conservation property is given in this section, those of linear and angular momentum are given in the appendix. Finally, in Section 5, a range of examples showing the great performance of the new integration method are presented. Comparison against the classical midpoint demonstrates the long term stability of the method.

2. Kinematics and constitutive law

2.1. Kinematics and strain fields

In this section we give a short summary of the key ingredients of the theory and the most relevant quantities involved. For more details the reader is referred to the standard literature on solid and structural mechanics.

At a reference time t_0 we consider a configuration of the body given by the vectors $\mathbf{X}$. Its corresponding placement at time t is denoted $\mathbf{x}$. The deformation gradient is then defined as $\mathbf{F} = \partial\mathbf{x}/\partial\mathbf{X}$. All centre points of the rod cross sections define the centre line, which, section-wise, is assumed to be smooth. An arc length parametrisation of this line is given by the arc length at the reference configuration s. A Cartesian coordinate system of bases $\mathbf{e}_i$, $i = 1, 2, 3$ is introduced. The deformations are restricted to the plane $\mathbf{e}_1 - \mathbf{e}_2$. Beside the Cartesian coordinates, a suitable convected curvilinear coordinate system is defined by the triple s, z, x_3 with z being the coordinate in the direction of the normal vector. Let's define $\mathbf{X}_0$ as the placement of the centre line at the reference configuration. One has $\mathbf{X}(s, z) = \mathbf{X}_0(s) + z\mathbf{N}(s)$, where $\mathbf{N}$ is the normal vector in the cross section. The cross section is assumed to exhibit a rotation which is defined by the tensor

$$\mathbf{R} = \cos\gamma\left(\mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2\right) - \sin\gamma\left(\mathbf{e}_1 \otimes \mathbf{e}_2 - \mathbf{e}_2 \otimes \mathbf{e}_1\right), \tag{1}$$

where γ is the rotation angle of the cross section. Following the Timoshenko kinematics, the deformed cross sections are assumed to remain in a plane but shear deformation is considered. Accordingly, the actual placement can be completely characterized by

$$\mathbf{x} = \mathbf{X}(s) - z\mathbf{N}(s) + \mathbf{u}(s) + z\mathbf{R}\mathbf{N} = \mathbf{X}_0(s) + \mathbf{u}(s) + z\mathbf{R}\mathbf{N}. \tag{2}$$

At the reference configuration a local bases can be defined as: $\mathbf{G}_1 = \mathbf{X}_{,s} = \mathbf{X}_{0,s} + z\mathbf{N}_{,s}$, $\mathbf{G}_2 = \mathbf{N}$ and $\mathbf{G}_3 = \mathbf{e}_3$. The corresponding tangent vectors in the actual configuration are given as $\mathbf{g}_1 = \mathbf{x}_{,s} = \mathbf{X}_{0,s} + \mathbf{u}_{,s} + z\left(\mathbf{R}\mathbf{N}\right)_{,s}$, $\mathbf{g}_2 = \mathbf{x}_{,z} = \mathbf{R}\mathbf{N}$ and $\mathbf{g}_3 = \mathbf{e}_3$. The right