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Nonlinear dynamics of shells conveying pulsatile flow with pulse-wave propagation. Theory and numerical results for a single harmonic pulsation

Eleonora Tubaldi, Marco Amabili*, Michael P. Païdoussis

Department of Mechanical Engineering, McGill University, 817 Sherbrooke Street West, Montreal, Canada H3A 0C3

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A B S T R A C T

In deformable shells conveying pulsatile flow, oscillatory pressure changes cause local movements of the fluid and deformation of the shell wall, which propagate downstream in the form of a wave. In biomechanics, it is the propagation of the pulse that determines the pressure gradient during the flow at every location of the arterial tree. In this study, a woven Dacron aortic prosthesis is modelled as an orthotropic circular cylindrical shell described by means of the Novozhilov nonlinear shell theory. Flexible boundary conditions are considered to simulate connection with the remaining tissue. Nonlinear vibrations of the shell conveying pulsatile flow and subjected to pulsatile pressure are investigated taking into account the effects of the pulse-wave propagation. For the first time in literature, coupled fluid-structure Lagrange equations of motion for a non-material volume with wave propagation in case of pulsatile flow are developed. The fluid is modeled as a Newtonian inviscid pulsatile flow and it is formulated using a hybrid model based on the linear potential flow theory and considering the unsteady viscous effects obtained from the unsteady time-averaged Navier-Stokes equations. Contributions of pressure and velocity propagation are also considered in the pressure drop along the shell and in the pulsatile frictional traction on the internal wall in the axial direction. A numerical bifurcation analysis employs a refined reduced order model to investigate the dynamic behavior of a pressurized Dacron aortic graft conveying blood flow. A pulsatile time-dependent blood flow model is considered by applying the first harmonic of the physiological waveforms of velocity and pressure during the heart beating period. Geometrically nonlinear vibration response to pulsatile flow and transmural pulsatile pressure, considering the propagation of pressure and velocity changes inside the shell, is here presented via frequency-response curves, time histories, bifurcation diagrams and Poincaré maps. It is shown that traveling waves of pressure and velocity cause a delay in the radial displacement of the shell at different values of the axial coordinate. The effect of different pulse wave velocities is also studied. Comparisons with the corresponding ideal case without wave propagation (i.e. with the same pulsatile velocity and pressure at any point of the shell) are here discussed. Bifurcation diagrams of Poincaré maps obtained from direct time integration have been used to study the system in the spectral neighborhood of the fundamental natural frequency. By in-

* Correspondence to: Department of Mechanical Engineering, McGill University, Macdonald Engineering Building, 817 Sherbrooke Street West, Montreal, PQ, Canada H3A 0C3.
E-mail address: marco.amabili@mcgill.ca (M. Amabili).

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creasing the forcing frequency, the response undergoes very complex nonlinear dynamics (chaos, amplitude modulation and period-doubling bifurcation), here deeply investigated.

1. Introduction

In deformable shells conveying pulsatile flow, pulsating pressure and flow propagate downstream in the form of progressive waves at the same wave speed. Lamb [1] considered for the first time in literature the problem of "the velocity of sound in a tube, as affected by the elasticity of the walls", thus involving both wave propagation and coupling between motions of the compressible fluid and the elastic tube wall. Lamb's work has been extended by numerous researchers dealing with the problem of wave-propagation in fluid-filled cylinders [2–4]. Unsteady flow characteristics and wave propagation through elastic tubes have also been studied more recently by Zamir [5].

In biomechanics, the pulsating flow of blood in the arteries causes wave propagation in the vessel walls and it is the propagation of the pulse that determines the pressure gradient during the flow at every location of the arterial tree. Womersley [6] was one of the first to experimentally study pulsatile flow performing his studies on the femoral artery of a dog. In these studies, it was the pressure gradient that was used to determine the flow characteristics indirectly. The interaction between the fluid and the vessel walls depends mostly on the physical-mechanical properties of the arterial tissues and the blood. In particular, the propagation velocity of pulse waves through the arteries is a means of diagnosing atherosclerotic arterial damage and determining the arterial tonus. The arterial pulse wave velocity (PWV) has been shown to be related to the underlying wall stiffness through the Moens-Korteweg [7] equation and has been used in a variety of applications for noninvasive estimation of arterial stiffness [8]. Taylor [9] showed that the presence of reflected waves causes the measured transmission velocity of a harmonic wave to vary greatly with frequency. Using the technique of measuring wave front velocities with a delay line (McDonald [10]), Nichols and McDonald [11] made an extensive study of the wave velocity in the ascending aorta of dogs, showing that phase velocity values, averaged over the first ten harmonics, were in close agreement with the velocity of the wave front. Their results also demonstrated that an increase in mean arterial pressure increases the pulse wave velocity.

Modeling three dimensional blood flow in compliant arteries is extremely challenging because of the complexity of solving the coupled blood flow/vessel deformation problem. For this reason in some studies (Taylor et al. [12] and Oshima et al. [13]), the rigid-wall approximation of the vessel is justified because the vessel diameter change during the cardiac cycle is observed to be approximately 5–10% in most of the major arteries; moreover, in diseased vessels, the arteries are even less compliant and, wall motion is further reduced. Perktold and Rappitsch [14] showed that in the case of carotid artery under normal conditions, wall deformability does not significantly alter the velocity field. They demonstrated the effect of wall distensibility on the flow and wall shear stress patterns by comparing with the results of a rigid wall model. In particular, they found that the rigid wall model agrees with the diastolic geometry at the end of the pulse period of the compliant model. Moreover, the flow rates at the common carotid inflow and at the external outflow were found to be equal in both cases. However, this holds true for arteries with small wall motion and may not be valid for arteries with larger wall deformation [15] (e.g. the thoracic aorta). In particular, assuming rigid vessel walls means neglecting the wave propagation phenomenon within the tube and consequently changing the character of the resultant solutions. For the analysis of flow in compliant vessels, Formaggia et al. [16] proposed an approach to couple three-dimensional domains of the original Navier-Stokes equations with a convenient one-dimensional domain used to describe wave propagation methods. In order to properly represent the propagation phenomenon due to the fluid-structure interaction and not to fluid compressibility, the 2D/3D fluid-structure problem has been coupled with a reduced one-dimensional model, which acts as an "absorbing" device for the waves exiting the computational domain. The appropriate framework for solving problems of computational modeling of blood flow in deforming vessels is the arbitrary Lagrangian-Eulerian (ALE) description of continuous media, in which the fluid and solid domains are allowed to move to follow the distensible vessels and the deforming fluid domain [12,17]. The ALE approach has been employed, resulting in numerical models with a large number of degrees of freedom for developing realistic anatomic and physiologic models of the cardiovascular system. Figueroa et al. [15] developed a method for simulating blood flow in three-dimensional deformable models of arteries based on the coupling of the equations of the deformation of the vessel wall at the variational level as a boundary condition for the fluid domain, by using basic assumptions of a thin-walled structure. The computational effort in their method is comparable to that for rigid wall formulations while respecting the essential physics and enabling realistic simulation of wave-propagation phenomenon in the arterial system, as well as a linearized description of the wall deformation. Recently, Amabili et al. [18] investigated the stability of a straight aorta segment conveying blood flow using a numerical bifurcation analysis that employs a refined reduced-order model. In particular, they identified for the first time the nonlinear buckling (collapse) of the aorta as a possible reason behind the appearance of high stress regions at the inner layer of the aortic wall that may be responsible for the initiation of aortic dissection. A geometrically nonlinear shell theory that takes into account three anisotropic layers (intima, media and adventitia) and incompressible potential flow were used in the model. Virtually, the mechanics of all

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